UNIT I
DC & AC CIRCUITS

DC Circuits: Electrical circuit elements (R, L and C), Ohm's Law and its limitations, KCL & KVL, series, parallel, series-parallel circuits, Super Position theorem, Simple numerical problems.
AC Circuits: A.C. Fundamentals: Equation of AC Voltage and current, waveform, time period, frequency, amplitude, phase, phase difference, average value, RMS value, form factor, peak factor, Voltage and current relationship with phasor diagrams in R, L, and C circuits, Concept of Impedance, Active power, reactive power and apparent power, Concept of power factor (Simple Numerical problems).

DC CIRCUITS

INTRODUCTION:

i) Charge: (Q or q)

An electric charge occurs when the atoms of matter contain unequal numbers of electrons and protons. Protons are positively charged and electrons are negatively charged.

The net charge is equal to the sum of all the charges in an atom. Since it is so difficult to count them all, an ideal number, known as the **coulomb**, is used to represent a large number of elementary charges. This is approximately 6.25 x 10^{18} elementary charges.

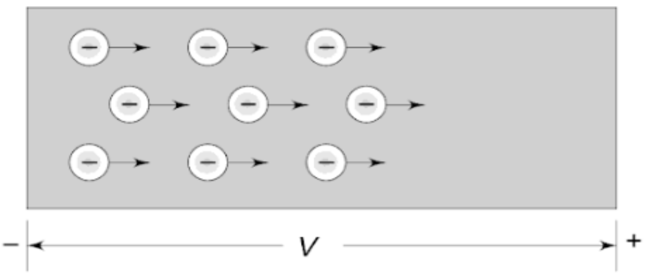
One Coulomb = 6.25 x 10^{18} electrons

Unit of charge(Q): Coulomb (C)

ii) Current: (I or i)

There are free electrons available in all semi-conductive and conductive materials. These free electrons move at random in all directions within the structure in the absence of external pressure or voltage.

If a certain amount of voltage is applied across the material, all the free electrons move in one direction depending on the polarity of the applied voltage, as shown in Fig.



This movement of electrons from one end of the material to the other end constitutes an electric current, denoted by either I or i. The conventional direction of current flow is opposite to the flow of -ve charges, i.e. the electrons.

Current is defined as the rate of flow of electrons in a conductive or semi-conductive material. It is measured by the number of electrons that flow past a point in unit time. Expressed mathematically,

$$I = \frac{Q}{t}$$

Where I is the current, Q is the charge of electrons, and t is the time, or

AVRK

$$i = \frac{dq}{dt}$$

Where dq is the small change in charge, and dt is the small change in time.

Unit of Current : Ampere(A)

iii) Voltage (or) Potential Difference (or) Potential (or) emf: (V)

According to the structure of an atom, we know that there are two types of charges: positive and negative. A force of attraction exists between these positive and negative charges. A certain amount of energy (work) is required to overcome the force and move the charges through a specific distance. All opposite charges possess certain amount of potential energy because of the separation between them. The difference in potential energy of the charges is called the potential difference.

Potential difference in electrical terminology is known as voltage(pressure), and is denoted either by V or v. It is expressed in terms of energy (W) per unit charge (Q); i.e.

$$V = \frac{W}{Q} \quad \text{or} \quad v = \frac{dw}{dq}$$

where dw is the small change in energy, and dq is the small change in charge.

Unit of Voltage : Volt(V)

iv) POWER(P) & ENERGY(W):

Energy is the capacity for doing work, i.e. energy is nothing but stored work. Energy may exist in many forms such as mechanical, chemical, electrical and so on. (OR) **Work is done whenever a force moves something over a distance.**

Work Done, W = Force * Distance = F * S

Power is the rate of change of energy (the rate at which work is done) and is denoted by either P or p. If certain amount of energy is used over a certain length of time, then

$$Power = \frac{Energy}{Time} = \frac{W}{t} \quad \text{or} \quad p = \frac{dW}{dt}$$

where 'dw' is the change in energy and 'dt' is the change in time.

$$p = \frac{dW}{dt} = \frac{dW}{dq} * \frac{dq}{dt}$$

$$p = vi$$

Unit of Energy : Joule(J)
Unit of Power : Watt(W)

$$\mathbf{P = VI = I^2R =} \frac{V^2}{R}$$

AVRK

NOTE: MULTIPLES & SUB-MULTIPLES USED

Multiple	Prefix	Symbol
10^{24}	yotta	Y
10^{21}	zetta	Z
10^{18}	exa	E
10^{15}	peta	P
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10^{1}	deca	da

Submultiple	Prefix	Symbol
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a
10^{-21}	zepto	z
10^{-24}	yocto	y

ELECTRICAL CIRCUIT ELEMENTS

1) RESISTANCE:

When a current flows in a material, the free electrons move through the material and collide with other atoms. These collisions cause the electrons to lose some of their energy. This loss of energy per unit charge is the drop in potential across the material. The amount of energy lost by the electrons is related to the physical property of the material. These collisions restrict the movement of electrons.

The property of a material to restrict the flow of electrons is called resistance, denoted by R. The symbol for the resistor is shown in Fig.

The unit of resistance is ohm (Ω). Ohm is defined as the resistance offered by the material when a current of one ampere flows between two terminals with one volt applied across it.

According to Ohm's law, the current is directly proportional to the voltage and inversely proportional to the total resistance of the circuit, i.e.

$$I = \frac{V}{R} \quad \text{or} \quad i = \frac{v}{R}$$

We can write the above equation in terms of charge as follows.

$$V = R\frac{dq}{dt}, \quad \text{or} \quad i = \frac{v}{R} = Gv$$

where ‘G’ is the conductance of a conductor. The units of resistance and conductance are ohm (Ω) and mho (℧) respectively.

AVRK

When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat. The power absorbed by the resistor is given by

$$P = vi = (iR)i = i^2 R$$

where 'i' is the current in the resistor in amps, and 'v' is the voltage across the resistor in volts. Energy lost in a resistance in time 't' is given by

$$W = \int_0^t pdt = pt = i^2Rt = \frac{v^2}{R}t$$

where v is the volts, R is in ohms, t is in seconds and W is in joules.

Resistance of a material is directly proportional to its length and inversely proportional to the area of cross-section.

$$R \propto \frac{l}{A}$$

$$\therefore R = \rho \frac{l}{A}$$

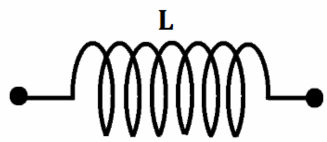
where ρ is constant of proportionality and is known as **resistivity** or **specific resistance,** l is length of material and A is cross sectional area of material.

2) INDUCATNCE:

A wire of certain length, when twisted into a coil becomes a basic inductor. If current is made to pass through an inductor, an electromagnetic field is formed. A change in the magnitude of the current changes the electromagnetic field.

Increase in current expands the fields and decrease in current reduces it. Therefore, a change in current produces change in the electromagnetic field, which induces a voltage across the coil according to Faraday's law of electromagnetic induction.

The unit of inductance is Henry, denoted by H. The inductance is one Henry when current through the coil, changing at the rate of one ampere per second, induces one volt across the coil. The symbol for inductance is shown in Fig.



The current-voltage relation is given by

$$v = L\frac{di}{dt}$$

where 'v' is the voltage across inductor in volts, and 'i' is the current through inductor in amps. We can rewrite the above equations as

AVRK

$$di = \frac{1}{L} v dt$$

Integrating both sides, we get

$$\int_0^t di = \frac{1}{L} \int_0^t v dt$$

$$i(t) - i(0) = \frac{1}{L} \int_0^t v dt$$

$$i(t) = \frac{1}{L} \int_0^t v dt + i(0)$$

From the above equation we note that the current in an inductor is dependent upon the integral of the voltage across its terminals and the initial current in the coil, $i(0)$.

The power absorbed by inductor is

$$P = vi = Li \frac{di}{dt} \text{ watts}$$

The energy stored by the inductor is

$$W = \int_0^t p dt$$

$$= \int_0^t Li \frac{di}{dt} dt = \frac{Li^2}{2}$$

From the above discussion, we can conclude the following.

i) The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.

ii) A small change in current within zero time through an inductor gives an infinite voltage across the inductor, which is physically impossible. In a fixed inductor the current cannot change abruptly.

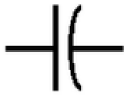
iii) The inductor can store finite amount of energy, even if the voltage across the inductor is zero.

iv) A pure inductor never dissipates energy, only stores it. That is why it is also called a non-dissipative passive element. However, physical inductors dissipate power due to internal resistance.

3) CAPACITANCE:

Any two conducting surfaces separated by an insulating medium exhibit the property of a capacitor. The conducting surfaces are called electrodes, and the insulating medium is called dielectric. A capacitor stores energy in the form of an electric field that is established by the opposite charges on the two electrodes. The electric field is represented by lines of force between the positive and negative charges and is concentrated within the dielectric. The amount of charge per unit voltage that is capacitor can store is its capacitance, denoted by C.

The unit of capacitance is Farad, denoted by F. One Farad is the amount of capacitance when one coulomb of charge is stored with one volt across the plates. The symbol for capacitance is shown in Fig.



A capacitor is said to have greater capacitance if it can store more charge per unit voltage and the capacitance is given by

$$C = \frac{Q}{V}, \quad \text{or} \quad C = \frac{q}{v}$$

We can write the above equation in terms of current as

$$i = C\frac{dv}{dt} \qquad \left(\because \; i = \frac{dq}{dt}\right)$$

where v is the voltage across capacitor, i is the current through it

$$dv = \frac{1}{C}\, idt$$

Integrating both sides, we have

$$\int_0^t dv = \frac{1}{C}\int_0^t idt$$

$$v(t) - v(0) = \frac{1}{C}\int_0^t idt$$

$$v(t) = \frac{1}{C}\int_0^t idt + v(0)$$

where $v(0)$ indicates the initial voltage across the capacitor.

From the above equation, the voltage in a capacitor is dependent upon the integral of the current through it, and the initial voltage across it.

The power absorbed by the capacitor is given by

$$p = vi = vC\,\frac{dv}{dt}$$

The energy stored by the capacitor is

$$W = \int_0^t p\,dt = \int_0^t vC\,\frac{dv}{dt}\,dt$$

$$W = \frac{1}{2}\,Cv^2$$

From the above discussion we can conclude the following

i) The current in a capacitor is zero if the voltage across it is constant; that means, the capacitor acts as an open circuit to dc.

ii) A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible. In a fixed capacitance the voltage cannot change abruptly.

iii) The capacitor can store a finite amount of energy, even if the current through it is zero.

iv) A pure capacitor never dissipates energy, but only stores it; that is why it is called non-dissipative passive element. However, physical capacitors dissipate power due to internal resistance.

V–I relation of circuit elements

Circuit element	*Voltage (V)*	*Current (A)*	*Power (W)*
Resistor R (Ohms Ω)	$v = Ri$	$i = \frac{v}{R}$	$P = i^2 R$
Inductor L (Henry H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + i_0$ where i_0 is the initial current in inductor	$P = L\,i\frac{di}{dt}$
Capacitor C (Farad F)	$v = \frac{1}{C}\int i\,dt + v_0$ where v_0 is the initial voltage across capacitor	$i = c\frac{dv}{dt}$	$P = cv\frac{dv}{dt}$

OHM'S LAW

Ohm's law deals with the relationship between current, voltage and ideal resistance. This relationship was introduced by German physicist George Simon Ohm. That is why the law is well known as Ohm's law.

Statement:

At constant temperature, the current through an ideal resistor is directly proportional to the voltage applied across the resistor.

$$I \propto V \Leftrightarrow V \propto I$$

The constant of proportionality is written as R and this is the resistance value of the resistor.

$$V = RI$$

The main criteria for Ohm's law are to keep the resistance constant because proportionality constant in the relationship is resistance R. But we know that the variation of temperature affects the value of resistance so to keep the resistance constant during experiments of Ohm's law the temperature is considered constant.

Applications of Ohm's Law:

There are thousands of applications of this law in our daily life. We will show only a few of them in this article.

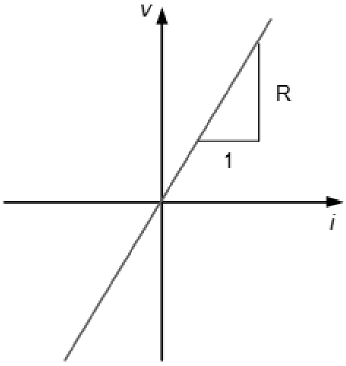
- Conventional Domestic Fan Regulator is one very common device where the current through the fan gets regulated by controlling the resistance of the regulator circuit.
- In voltage divider circuit this law is used to divide source voltage across the output resistance.
- In electronic circuits, there are many purposes where intentional voltage drop is required to supply specific voltage across different electronic elements. This is done by applying Ohm's law.
- In mainly dc ammeter and other dc measuring instruments shunt is used to divert current. Here also Ohm's law is used.

Limitation of Ohm's Law:

The limitations of Ohm's law are explained as follows:

- This law cannot be applied to unilateral networks. A unilateral network has unilateral elements like diode, transistors, etc., which do not have same voltage current relation for both directions of current.
- Ohm's law is also not applicable for non – linear elements. Non-linear elements are those which do not have current exactly proportional to the applied voltage, that means the resistance value of those elements changes for different values of voltage and current. Examples of non – linear elements are thyristor, electric arc, etc.

Ohm's law can also be viewed as a voltage-current characteristic for the resistor. Often, the voltage-current characteristic is presented graphically; in graphical form, the voltage-current characteristic is called the i-v curve. The i-v curve for a resistor is displayed in Fig.



KIRCHHOFF'S LAWS

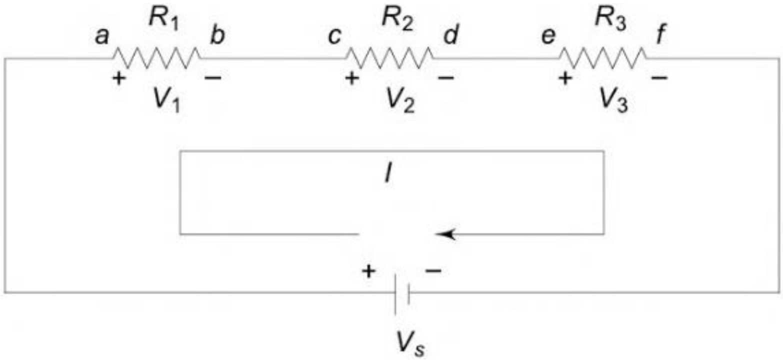
There are some simple relationships between currents and voltages of different branches of an electrical circuit. These relationships are determined by some basic laws that are known as Kirchhoff laws or more specifically Kirchhoff Current and Voltage laws. These laws are very helpful in determining the equivalent electrical resistance or impedance (in case of AC) of a complex network and the currents flowing in the various branches of the network. These laws are first derived by Guatov Robert Kirchhoff and hence these laws are also referred as Kirchhoff Laws.

1) KIRCHHOFF'S VOLTAGE LAW (KVL)

Statement:

Kirchhoff's voltage law states that the algebraic sum of all branch voltages around any closed path in a circuit is always zero at all instants of time.

When the current passes through a resistor, there is a loss of energy and, therefore, a voltage drop. In any element, the current always flows from higher potential to lower potential. Consider the circuit in Fig. It is customary to take the direction of current I as indicated in the figure, i.e. it leaves the positive terminal of the voltage source and enters into the negative terminal.



As the current passes through the circuit, the sum of the voltage drop around the loop is equal to the total voltage in that loop. Here the polarities are attributed to the resistors to indicate that the voltages at points a, c and e are more than the voltages at b, d and f, respectively, as the current passes from a to f.

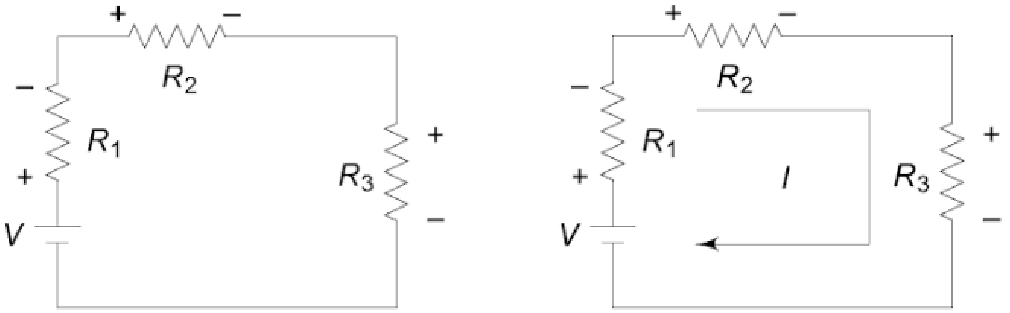
$$V_S - V_1 - V_2 - V_3 = 0$$

$$\therefore \quad V_s = V_1 + V_2 + V_3$$

AVRK

Consider the problem of finding out the current supplied by the source V in the circuit shown in Fig.

Our first step is to assume the reference current direction and to indicate the polarities for different elements. (See Fig.).



By using Ohm's law, we find the voltage across each resistor as follows.

$$V_{R1} = IR_1,\ V_{R2} = IR_2,\ V_{R3} = IR_3$$

where V_{R1}, V_{R2} and V_{R3} are the voltages across R_1, R_2 and R_3, respectively. Finally, by applying Kirchhoff's law, we can form the equation

$$V = V_{R1} + V_{R2} + V_{R3}$$

$$V = IR_1 + IR_2 + IR_3$$

From the above equation the current delivered by the source is given by

$$I = \frac{V}{R_1 + R_2 + R_3}$$

2) <u>KIRCHHOFF'S CURRENT LAW</u> (KVL)

Statement:

Kirchhoff's current law states that the sure of the currents entering any node is equal to the sum of the currents leaving that node.

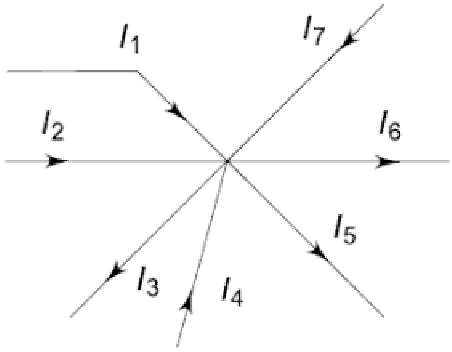
The node may be an interconnection of two or more branches. In any parallel circuit, the node is a junction point of two or more branches. The total current entering into a node is equal to the current leaving that node.

For example, consider the circuit shown in Fig., which contains two nodes A and B. The total current I_T entering node A is divided into I_1, I_2 and I_3. These currents flow out of node A.

According to Kirchhoff's current law, the current into node A is equal to the total current out of node A: that is, $I_T = I_1 + I_2 + I_3$. If we consider node B, all three currents I_1, I_2 and I_3 are entering B, and the total current I_T is leaving node B, Kirchhoff's current law formula at this node is therefore the same as at node A.

$$I_1 + I_2 + I_3 = I_T$$

In general, sum of the currents entering any point or node or junction equal to sum of the currents leaving from that point or node or junction as shown in Fig.



$$I_1 + I_2 + I_4 + I_7 = I_3 + I_5 + I_6$$

If all of the terms on the right side are brought over to the left side, their signs change to negative and a zero is left on the right side, i.e.

$$I_1 + I_2 + I_4 + I_7 - I_3 - I_5 - I_6 = 0$$

This means that the algebraic sum of all the currents meeting at a junction is equal to zero.

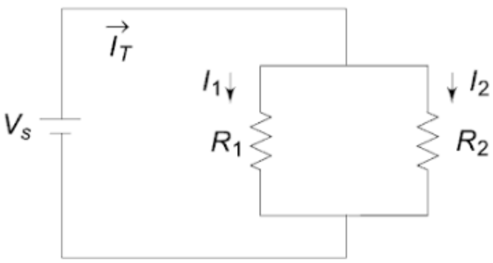
i) Current Division:

In a parallel circuit, the current divides in all branches. Thus, a parallel circuit acts as a current divider. The total current entering into the parallel branches is divided into the branch's currents according to the resistance values. The branch having higher resistance allows lesser current, and the branch with lower resistance allows more current. Let us find the current division in the parallel circuit shown in Fig.

The voltage applied across each resistor is Vs. The current passing through each resistor is given by

$$I_1 = \frac{V_s}{R_1}, \ I_2 = \frac{V_s}{R_2}$$

If R_T is the total resistance, which is given by $R_1R_2/(R_1 + R_2)$,



$$\text{Total current } I_T = \frac{V_s}{R_T} = \frac{V_s}{R_1 R_2}(R_1 + R_2)$$

$$\text{or } I_T = \frac{I_1 R_1}{R_1 R_2}(R_1 + R_2) \ \text{ since } V_s = I_1 R_1$$

$$I_1 = I_T \cdot \frac{R_2}{R_1 + R_2}$$

$$\text{Similarly,} \quad I_2 = I_T \cdot \frac{R_1}{R_1 + R_2}$$

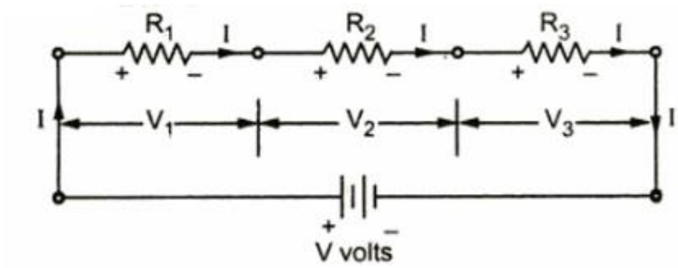
From the above equations, we can conclude that the current in any branch is equal to the ratio of the opposite branch resistance to the total resistance value, multiplied by the total current in the circuit. In general, if the circuit consists of 'm' branches, the current in any branch can be determined by

$$I_i = \frac{R_T}{R_i + R_T} I_T$$

where I_i represents the current in the ith branch
R_i is the resistance in the ith branch
R_T is the total parallel resistance to the ith branch and
I_T is the total current entering the circuit.

SERIES & PARALLEL REDUCTION TECHNIQUE:

a) Resistors in Series: [CURRENT SAME & VOLTAGE DIVISION]
Consider the resistances shown in the fig.



The resistances R_1, R_2 and R_3 are said to be in series. This combination is connected across a source of V volts. The current is flowing through all resistances and it is same indicated as 'I'.

Now let us study the **voltage distribution.**

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively

Then, $V = V_1 + V_2 + V_3$

Now according to Ohm's law, $V_1 = IR_1$, $V_2 = IR_2$, $V_3 = IR_3$

Current through all of them is same i.e. I

$\therefore$ $V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$

Applying Ohm's law to overall circuit,

$$V = I\ R_{eq}$$

where R_{eq} = Equivalent resistance of the circuit. By comparison of two equations,

$$R_{eq} = R_1 + R_2 + R_3$$

i.e. Total or **equivalent resistance** of the series circuit is arithmetic sum of the resistances connected in series.

For n resistances in series, $\boxed{R = R_1 + R_2 + R_3 + \ldots\ldots + R_n}$

AVRK

Characteristics of Series Circuits

1) The same current flows through each resistance.
2) The supply voltage V is the sum of the individual voltage drops across the resistances.

$$V = V_1 + V_2 + \ldots + V_n$$

3) The equivalent resistance is equal to the sum of the individual resistances.
4) The equivalent resistance is the largest of all the individual resistances.

i.e $\quad R > R_1 , R > R_2 , \ldots R > R_n$

NOTE:

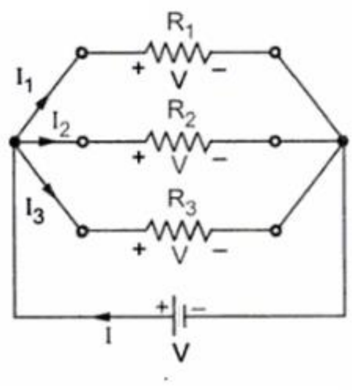
i) If the inductances $L_1, L_2, L_3, \ldots, L_n$ are in series, then the equivalent inductance is

$$L_{eq} = L_1 + L_2 + L_3 + \ldots + L_n$$

ii) If the capacitances $C_1, C_2, C_3, \ldots, C_n$ are in series, then the equivalent capacitance is

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots .. + \frac{1}{C_n}$$

b) Resistors in parallel: [VOLTAGE SAME & CURRENT DIVISION]



Consider a parallel circuit shown in the Fig.

In the parallel connection shown, the three resistances R_1, R_2 and R_3 are connected in parallel and combination is connected across a source of voltage 'V '.

In parallel circuit current passing through each resistance is different. Let total current drawn is say ' I ' as shown. There are 3 paths for this current, one through R_1, second through R_2 and third through R_3. Depending upon the values of R_1, R_2 and R_3 the appropriate fraction of total current passes through them. These individual currents are shown as I_1, I_2 and I_3. While the voltage across the two ends of each resistances R_1, R_2 and R_3 is the same and equals the supply voltage V.

Now let us study current distribution. Apply Ohm's law to each resistance.

AVRK

$$V = I_1 R_1 \ , \ V = I_2 R_2 \ , \ V = I_3 R_3$$

$$I_1 = \frac{V}{R_1} \ , \ I_2 = \frac{V}{R_2} \ , \ I_3 = \frac{V}{R_3}$$

$$I = I_1 + I_2 + I_3 = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right]$$

For overall circuit if Ohm's law is applied,

$$V = I R_{eq}$$

and $$I = \frac{V}{R_{eq}}$$

where R_{eq} = Total or **equivalent resistance** of the circuit

Comparing the two equations,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

where R is the equivalent resistance of the parallel combination.

In general if 'n' resistances are connected in parallel,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

Important result :

Now if n = 2, two resistances are in parallel then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

Characteristics of Parallel Circuits

1) The same potential difference gets across all the resistances in parallel.
2) The total current gets divided into the number of paths equal to the number of resistances in parallel. The total current is always sum of all the individual currents.

$$I = I_1 + I_2 + I_3 + \ldots\ldots + I_n$$

3) The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.

AVRK

NOTE:

i) If the inductances $L_1, L_2, L_3, \ldots, L_n$ are in parallel, then the equivalent inductance is

$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots .. + \frac{1}{L_n}$$

ii) If the capacitances $C_1, C_2, C_3, \ldots, C_n$ are in parallel, then the equivalent capacitance is

$$C_{eq} = C_1 + C_2 + C_3 + \ldots + C_n$$

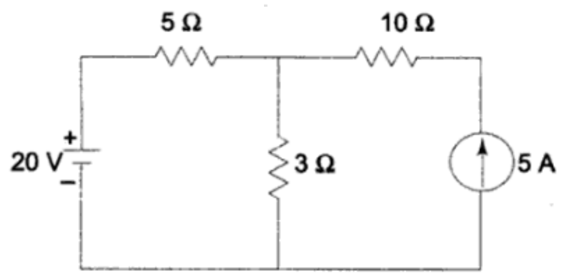
SUPERPOSITION THEOREM

Statement:

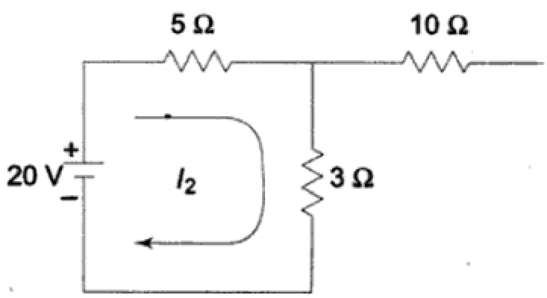
The superposition theorem states that in any linear network containing two or more sources, the response in any element is equal to the algebraic sum of the responses caused by individual sources acting alone, while the other sources are non-operative; that is, while considering the effect of individual sources, other ideal voltage sources and ideal current sources in the network are replaced by short circuit and open circuit across their terminals respectively. This theorem is valid only for linear systems.

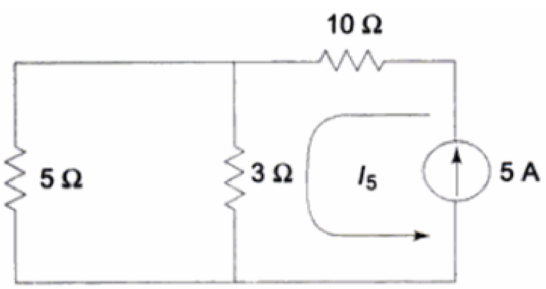
EX:

Consider the circuit which contains two sources as shown in Fig.



Now let us find the current passing through the 3 Ω resistor in the circuit. According to superposition theorem, the current I_2 due to the 20 V voltage source with 5 A source open circuited = 20/(5 + 3) = 2.5 A.





The current I_5 due to 5 A source with 20 V source short circuited is

$$I_5 = 5 \times \frac{5}{(3+5)} = 3.125 \text{ A}$$

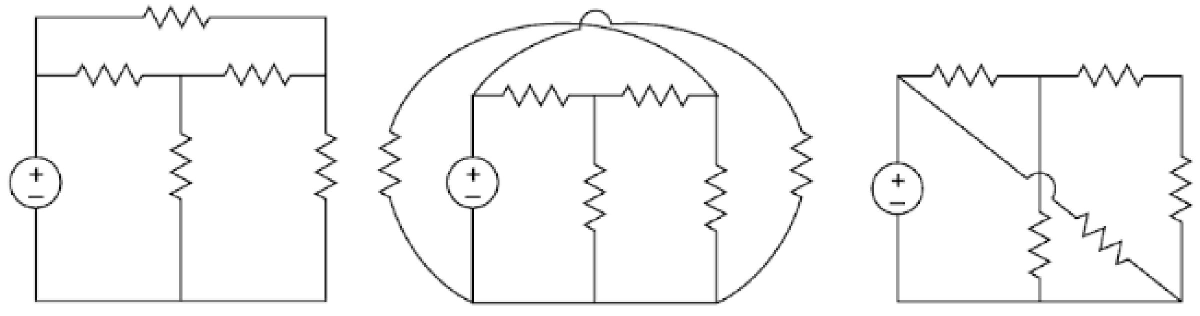
The total current passing through the 3 Ω resistor is

$$(2.5 + 3.125) = 5.625 \text{ A}$$

AVRK

MESH ANALYSIS

Mesh analysis is applicable only for planar networks. For non-planar circuits mesh analysis is not applicable. A circuit is said to be planar, if it can be drawn on a plane surface without crossovers. A non-planar circuit cannot be drawn on a plane surface without a crossover.



(a) (b) (c)

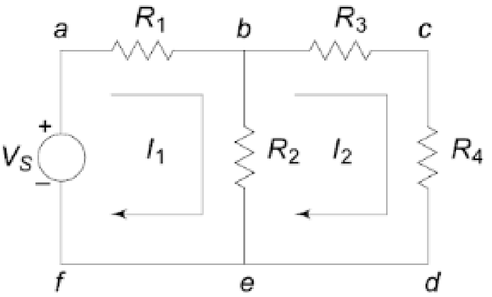
The Figure(a) is a planar circuit. Figure(b) is a non-planar circuit and Fig.(c) is a planar circuit which looks like a non-planar circuit. It has already been discussed that a loop is a closed path.

A **mesh** is defined as a loop which does not contain any other loops within it. To apply mesh analysis, our first step is to check whether the circuit is planar or not and the second is to select mesh currents. Finally, writing Kirchhoff's voltage law equations in terms of unknowns and solving them leads to the final solution.

EX:

Consider the circuit as shown in the Fig. It indicates that there are two loops 'abefa' and 'bcdeb' in the network. Let us assume loop currents I_1 and I_2 with directions as indicated in the figure.

Considering the loop abefa alone, we observe that current I_1 is passing through R_1 and (I_1-I_2) is passing through R_2. By applying Kirchhoff's voltage law, we can write



$$V_s = I_1R_1 + R_2(I_1 - I_2)$$

Similarly, if we consider the second mesh *bcdeb*, the current I_2 is passing through R_3 and R_4, and $(I_2 - I_1)$ is passing through R_2. By applying Kirchhoff's voltage law around the second mesh, we have

$$R_2(I_2 - I_1) + R_3 I_2 + R_4 I_2 = 0$$

By rearranging the above equations, the corresponding mesh current equations are

$$I_1(R_1 + R_2) - I_2 R_2 = V_s$$

$$-I_1 R_2 + (R_2 + R_3 + R_4) I_2 = 0$$

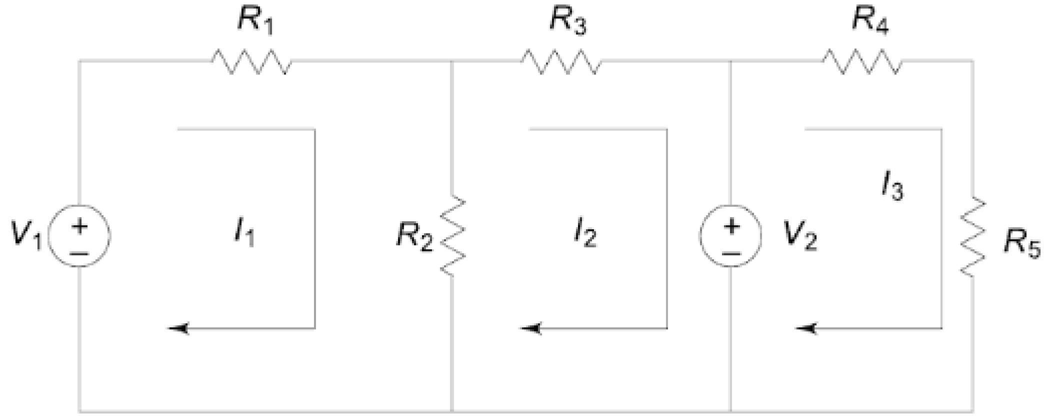
By solving the above equations, we can find the currents I_1 and I_2. If we observe Fig., the circuit consists of five branches and four nodes, including the reference node. **The number of mesh currents is equal to the number of mesh equations.**

And the number of equations = branches - (nodes -1).

In Fig., the required number of mesh currents would be 5 - (4 - 1) = 2.
In general, if we have 'B' number of branches and 'N' number of nodes including the reference node then the number of linearly independent mesh equations ,M = B - (N -1).

MESH EQUATIONS BY INSPECTION METHOD

The mesh equations for a general planar network can be written by inspection without going through the detailed steps. Consider a three mesh networks as shown in Fig.



The loop equations are

$$I_1R_1 + R_2(I_1 - I_2) = V_1$$

$$R_2(I_2 - I_1) + I_2R_3 = -V_2$$

$$R_4I_3 + R_5I_3 = V_2$$

Reordering the above equations, we have

$$(R_1 + R_2)\ I_1 - R_2I_2 = V_1$$

$$-R_2I_1 + (R_2 + R_3)I_2 = -V_2$$

$$(R_4 + R_5)I_3 = V_2$$

By comparing the above equations, the following observations can be taken into account.
a) The self resistance in each mesh.
b) The mutual resistances between all pairs of meshes and
c) The algebraic sum of the voltages in each mesh.

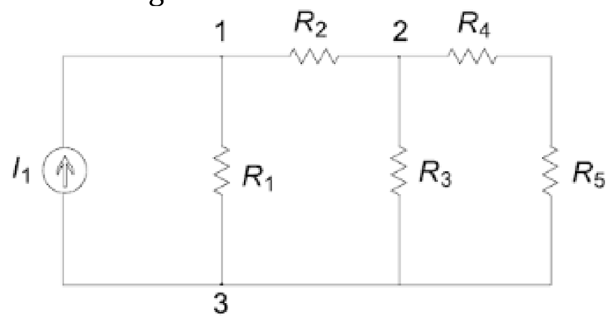
NODAL ANALYSIS

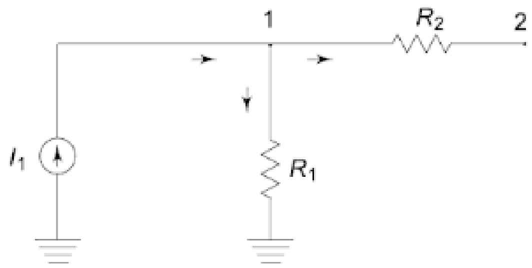
In general, in a N node circuit, one of the nodes is chosen as reference or datum node, then it is possible to write (N−1) nodal equations by assuming (N−1) node voltages.

For example, a 10 node circuit requires nine unknown voltages and nine equations. Each node in a circuit can be assigned a number or a letter. The node voltage is the voltage of a given node with respect to one particular node, called the reference node, which we assume at zero potential.

EX:

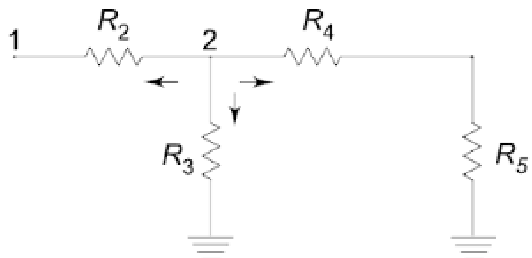
In the circuit shown in Fig., node 3 is assumed as the reference node. The voltage at node 1 is the voltage at that node with respect to node 3. Similarly, the voltage at node 2 is the voltage at that node with respect to node 3. Applying Kirchhoff's current law at node 1; the current entering is equal to the current leaving.





$$I_1 = \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2}$$

where V_1 and V_2 are the voltages at node 1 and 2, respectively. Similarly, at node 2, the current entering is equal to the current leaving as shown in Fig.



$$\frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4 + R_5} = 0$$

Rearranging the above equations, we have

$$V_1\left[\frac{1}{R_1} + \frac{1}{R_2}\right] - V_2\left[\frac{1}{R_2}\right] = I_1$$

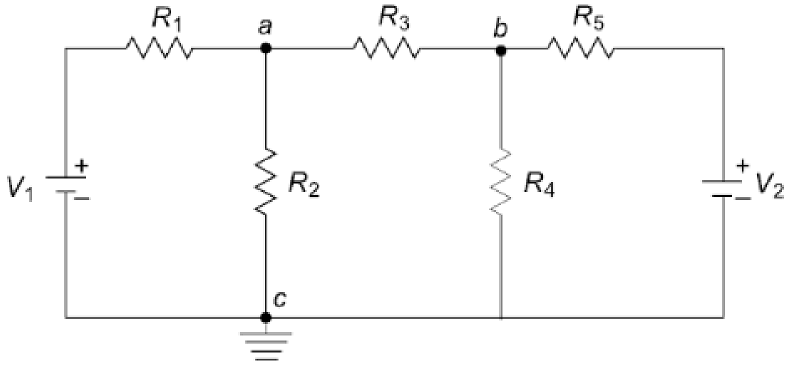
$$-V_1\left[\frac{1}{R_2}\right] + V_2\left[\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4 + R_5}\right] = 0$$

From the above equations, we can find the voltages at each node.

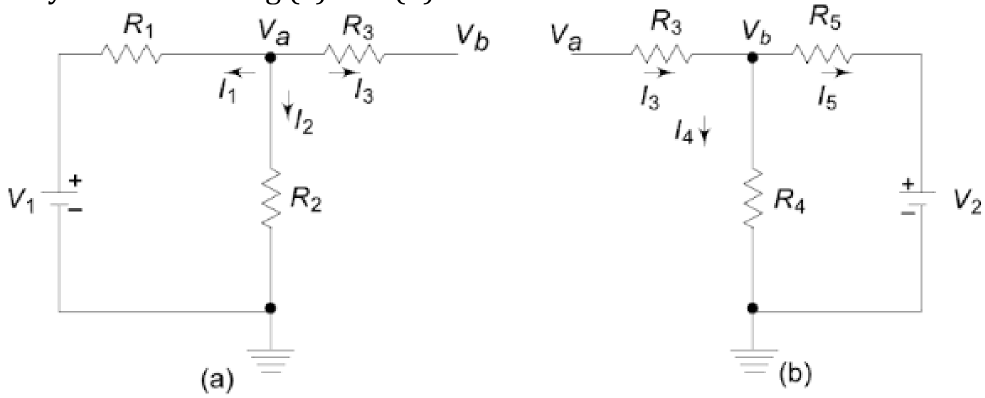
AVRK

Nodal Analysis by Inspection Method:

The nodal equations for a general planar network can also be written by inspection, without going through the detailed steps. Consider a three node resistive network, including the reference node, as shown in Fig.



In Fig., the points a and b are the actual nodes and c is the reference node. Now consider the nodes a and b separately as shown in Fig.(a) and (b).



In Fig. (a), according to Kirchhoff's current law, we have

$$I_1 + I_2 + I_3 = 0$$

$$\therefore \quad \frac{V_a - V_1}{R_1} + \frac{V_a}{R_2} + \frac{V_a - V_b}{R_3} = 0$$

In Fig (b), if we apply Kirchhoff's current law, we get

$$I_4 + I_5 = I_3$$

$$\therefore \quad \frac{V_b - V_a}{R_3} + \frac{V_b}{R_4} + \frac{V_b - V_2}{R_5} = 0$$

Rearranging the above equations, we get

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_a - \left(\frac{1}{R_3}\right)V_b = \left(\frac{1}{R_1}\right)V_1$$

$$\left(-\frac{1}{R_3}\right)V_a + \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)V_b = \frac{V_2}{R_5}$$

AVRK

In general, the above equations can be written as

$$G_{aa} V_a + G_{ab} V_b = I_1$$

$$G_{ba} V_a + G_{bb} V_b = I_2$$

By comparing Eqs we have the self conductance at node a, $G_{aa} = (1/R_1 + 1/R_2 + 1/R_3)$ is the sum of the conductances connected to node a. Similarly, $G_{bb} = (1/R_3 + 1/R_4 + 1/R_5)$, is the sum of the conductances connected to node b. $G_{ab} = (-1/R_3)$, is the sum of the mutual conductances connected to node a and node b. Here all the mutual conductances have negative signs. Similarly, $G_{ba} = (-1/R_3)$ is also a mutual conductance connected between nodes b and a. I_1 and I_2 are the sum of the source currents at node a and node b, respectively. The current which drives into the node has positive sign, while the current that drives away from the node has negative sign.

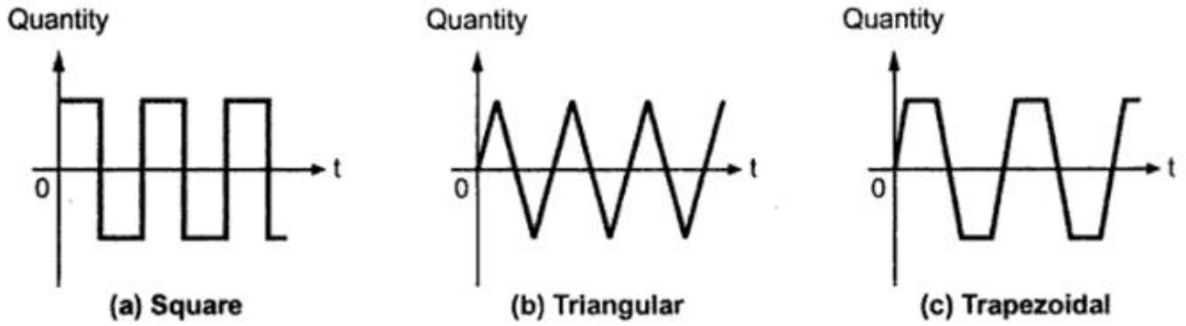
AC CIRCUITS

AC FUNDAMENTALS:

TYPES OF A.C. WAVEFORMS

The waveform of alternating voltage or current is shown purely sinusoidal in the Fig.(b). But, in practice, a quantity which undergoes variations in its instantaneous values, in magnitude as well as direction with respect to some zero reference is called an alternating quantity. The graph of such quantity against time is called its waveform. Various types of alternating waveforms other than sinusoidal are shown in the Fig.(a),(b) and (c).

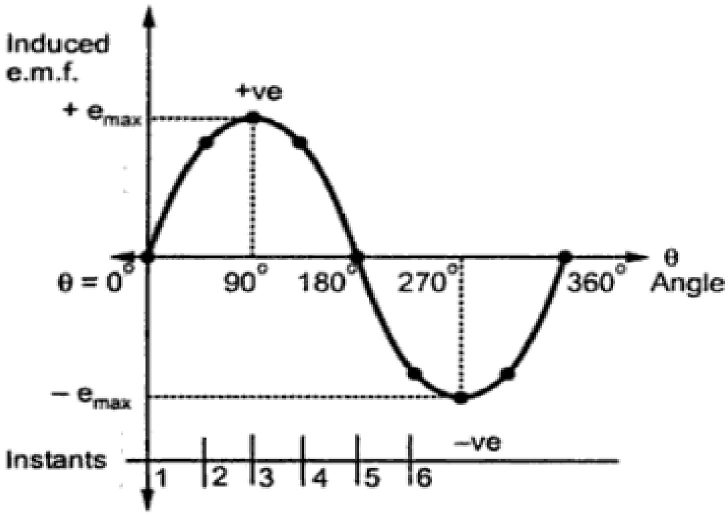
Out of all these types of alternating waveforms, purely sinusoidal waveform is preferred for a.c. system. There are few advantages of selecting purely sinusoidal as the standard waveform.



(a) Square (b) Triangular (c) Trapezoidal

GRAPHICAL REPRESENTATION OF THE SINEWAVE (INDUCED EMF)

The instantaneous values of the induced e.m.f. in any conductor, as it is rotated from θ = 0° to 360°, i.e. through one complete revolution can be represented as shown in the Fig. From the Fig., the waveform generated by the instantaneous values of the induced e.m.f. in any conductor is **purely sinusoidal** in nature.



EQUATION OF AC VOLTAGE (OR) CURRENT:

$$\theta = \omega t \quad \text{radians}$$

$$\boxed{e = E_m \sin(\omega t)}$$

$$\omega = 2\pi f \quad \text{rad / sec.}$$

$$\boxed{e = E_m \sin(2\pi f t)}$$

$$f = \frac{1}{T} \quad \text{seconds}$$

$$\boxed{e = E_m \sin\left(\frac{2\pi}{T} t\right)}$$

(or)

$$i = I_m \sin \omega t$$

$$i = I_m \sin 2\pi f t$$

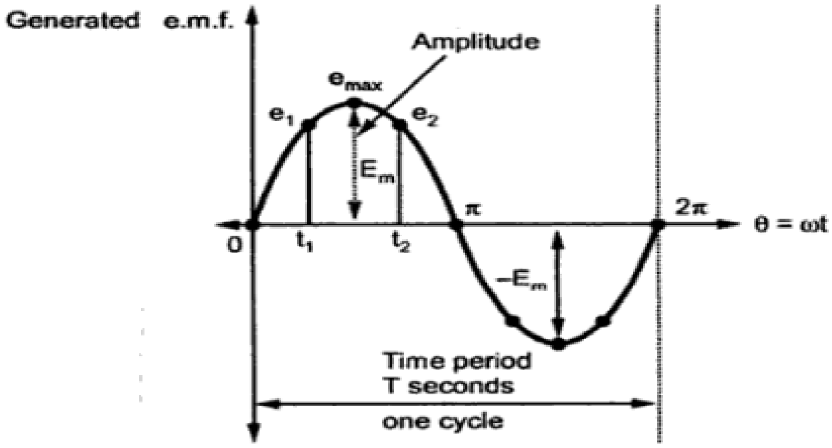
$$i = I_m \sin\left(\frac{2\pi t}{T}\right)$$

BASIC DEFINITIONS

i) Instantaneous Value:

The value of an alternating quantity at a particular instant is known as its instantaneous value.

EX: e1 and e2 are the instantaneous values of an alternating e.m.f. at the instants t1 and t2 respectively shown in the Fig.



ii) Waveform :

The graph of instantaneous values of an alternating quantity plotted against time is called its waveform.

iii) Cycle :

Each repetition of a set of positive and negative instantaneous values of the alternating quantity is called a cycle.

Such repetition occurs at regular interval of time. Such a waveform which exhibits variations that reoccur after a regular time interval is called **periodic waveform.**

For Periodic waveform,

$$f(t) = f(t+T)$$

A cycle can also be defined as that interval of time during which a complete set of non-repeating events or waveform variations occur (containing positive as well as negative loops). One such cycle of the alternating quantity is shown in the above Fig.

One cycle corresponds to 2Π radians or 360°.

iv) Time Period (T):

The time taken by an alternating quantity to complete its one cycle is known as its time period and denoted by T seconds. After every T seconds, the cycle of an alternating quantity repeats. This is shown in the above Fig.

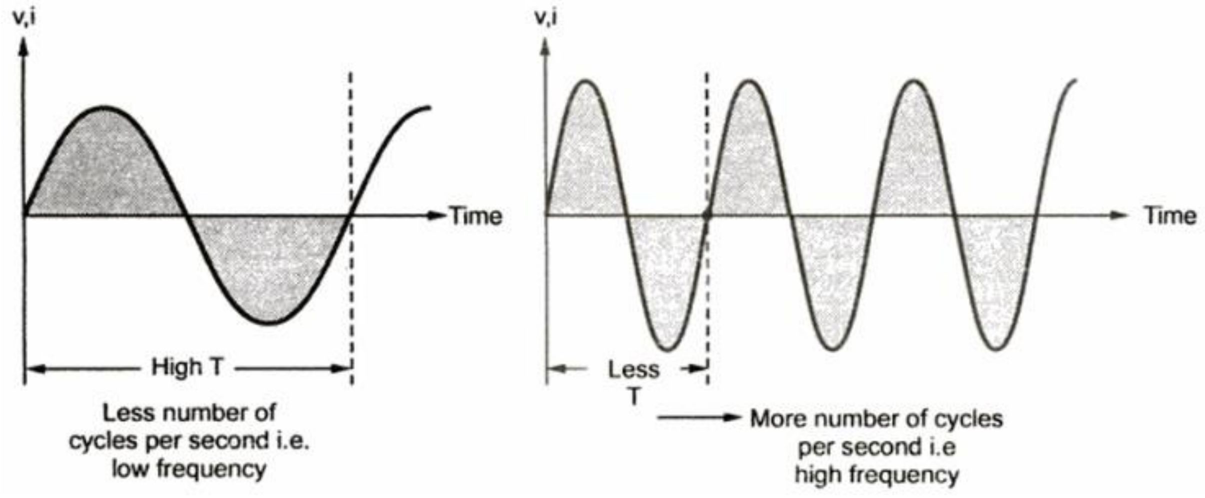
v) Frequency (f):

The number of cycles completed by an alternating quantity per second is known as its frequency. It is denoted by 'f' and it is measured in cycles / second which is known as Hertz, denoted as Hz. As time period T is time for one cycle i.e. seconds / cycle and frequency is cycles/second, we can say that frequency is reciprocal of the time period.

$$f = \frac{1}{T} \quad Hz$$

In India, The standard frequency = 50Hz

As time period increases, frequency decreases while as time period decreases, frequency increases. This is shown in the following Fig.



vi) Amplitude (or) Peak Value:

The maximum value attained by an alternating quantity during positive or negative half cycle is called its amplitude. It is denoted as Em or Im.

Thus Em is called peak value of the voltage while Im is called peak value of the current.

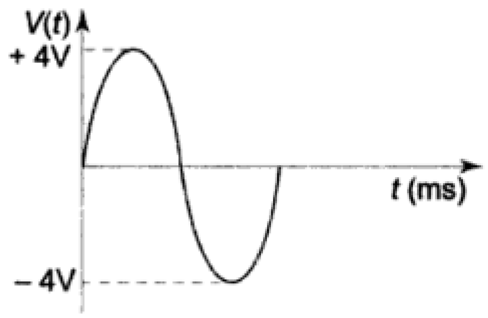
vii) Angular Frequency (ω):

It is the frequency expressed in electrical radians per second. As one cycle of an alternating quantity corresponds to 2Π radians, the angular frequency can be expressed as (2Π x cycles/sec.) It is denoted by 'ω' and its unit is radians/second. Now, cycles/ sec. mean frequency. Hence the relation between frequency 'f' and angular frequency 'ω' is,

$$\omega = 2\pi f \quad \text{radians/sec.} \quad \text{or} \quad \omega = \frac{2\pi}{T} \quad \text{radians/sec.}$$

viii) Peak to Peak value: (V_{PP})

The peak to peak value is the value from positive peak to the negative peak as shown in the fig. From the fig., peak to peak value is 8V.



AVERAGE VALUE OF PERIODIC WAVEFORM (or) SINE WAVEFORM

In general, the average value of any function v(t), with period T is given by

$$v_{av} = \frac{1}{T}\int_0^T v(t)\, dt$$

That means that the average value of a curve in the X- Y plane is the total area under the complete curve divided by the distance of the curve. **The average value of a sine wave over one**

complete cycle is always zero. So the average value of a sine wave is defined over a half-cycle, and not a full cycle period.

The average value of the sine wave is the total area under the half-cycle curve divided by the distance of the curve.

The average value of the sine wave

$v(t) = V_P \sin \omega t$ **is given by**

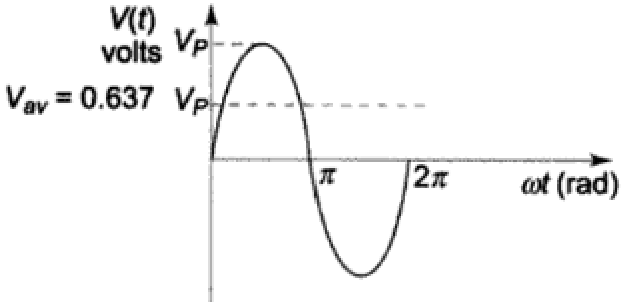
$$V_{av} = \frac{\text{Area under curve for half cycle}}{\text{Length of base over half cycle}}$$

$$= \frac{\int_0^{\pi} V\, d\theta}{\pi}$$

$$v_{av} = \frac{1}{\pi}\int_0^{\pi} V_P \sin \omega t \, d(\omega t)$$

$$= \frac{1}{\pi}\left[-V_P \cos \omega t\right]_0^{\pi}$$

$$= \frac{2V_P}{\pi} = 0.637\, V_P$$



Importance of Average Value:

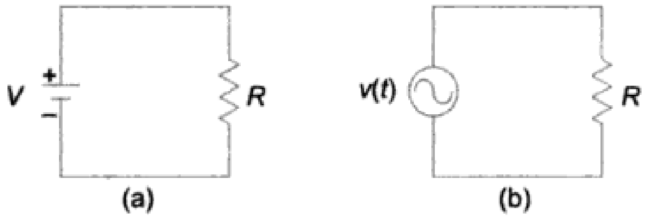
1. The average value is used for applications like battery charging.
2. The charge transferred in capacitor circuits is measured using average values.
3. The average values of voltages and currents play an important role in analysis of the rectifier circuits.
4. The average value is indicated by d.c. ammeters and voltmeters.
5. The average value of purely sinusoidal waveform is always zero.

ROOT MEAN SQUARE(RMS) VALUE OF PERIODIC WAVEFORM

(OR)

EFFECTIVE VALUE OF PERIODIC WAVEFORM (or) SINE WAVEFORM

The root mean square (rms) value of a sine wave is a measure of the heating effect of the wave. When a resistor is connected across a dc voltage source as shown in Fig.(a), a certain amount of heat is produced in the resistor in a given time. A similar resistor is connected across an ac voltage source for the same time as shown in Fig. (b). The value of the ac voltage is adjusted such that the same amount of heat is produced in the resistor as in the case of the dc source. This value is called the rms value.



That means the rms value of a sine wave is equal to the dc voltage that produces the same heating effect. In general, the rms value of any function with period T has an effective value given by

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T v(t)^2\, dt}$$

Consider a function $v(t) = V_P \sin \omega t$

The rms value, $$V_{rms} = \sqrt{\frac{1}{T}\int_0^T (V_P \sin \omega t)^2\, d(\omega t)}$$

$$= \sqrt{\frac{1}{T}\int_0^{2\pi} V_P^2 \left[\frac{1-\cos 2\omega t}{2}\right] d(\omega t)}$$

$$= \frac{V_P}{\sqrt{2}} = 0.707\, V_P$$

If the function consists of a number of sinusoidal terms, that is

$$v(t) = V_0 + (V_{c1} \cos \omega t + V_{c2} \cos 2\,\omega t + \cdots) + (V_{s1} \sin \omega t + V_{s2} \sin 2\,\omega t + \cdots)$$

The rms, or effective value is given by

$$V_{rms} = \sqrt{V_0^2 + \frac{1}{2}(V_{c1}^2 + V_{c2}^2 + \cdots) + \frac{1}{2}(V_{s1}^2 + V_{s2}^2 + \cdots)}$$

Importance of R.M.S. Value:

1. In case of alternating quantities, the r.m.s. values are used for specifying magnitudes of alternating quantities. The given values such as 230 V, 110 V are r.m.s. values of alternating quantities unless and otherwise specified to be other than r.m.s.

In practice, everywhere, r.m.s. values are used to analyze alternating quantities.

2. The ammeters and voltmeters record the r.m.s. values of current and voltage respectively.
3. The heat produced due to a.c. is proportional to square of the r.m.s. value of the current.

PEAK(CREST) FACTOR OF PERIODIC WAVEFORM

The peak factor of any waveform is defined as the ratio of the peak value of the wave to the rms value of the wave.

$$\text{Peak factor} = \frac{V_P}{V_{rms}}$$

$$\text{Peak factor of the sinusoidal waveform} = \frac{V_P}{V_P/\sqrt{2}} = \sqrt{2} = 1.414$$

FORM FACTOR OF PERIODIC WAVEFORM

Form factor of a waveform is defined as the ratio of rms value to the average value of the wave.

$$\text{Form factor} = \frac{rms\ \text{value of the wave}}{\text{Average value of the wave}}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{av}}$$

Form factor of a sinusoidal waveform can be found from the above relation.

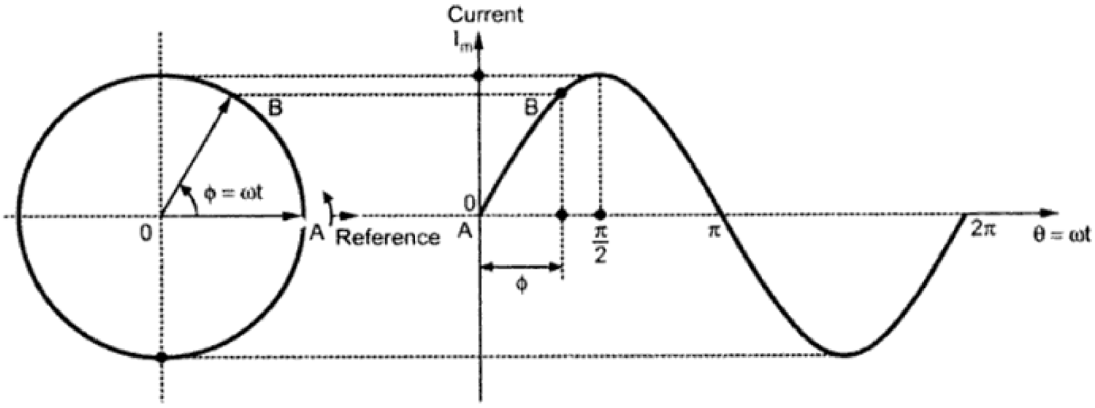
$$\text{For the sinusoidal wave, the form factor} = \frac{V_P/\sqrt{2}}{0.637\ V_P} = 1.11$$

PHASE AND PHASE DIFFERENCE

CONCEPT OF PHASE OF AN ALTERNATING QUANTITY

In the analysis of alternating quantities, it is necessary to know the position of the phasor representing that alternating quantity at a particular instant. It is represented in terms of angle θ in radians or degrees, measured from certain reference. Thus, phase can be defined as,

Phase: The phase of an alternating quantity at any instant is the angle Φ (in radians or degrees) travelled by the phasor representing that alternating quantity upto the instant of consideration, measured from the reference as shown in the fig.



Let X-axis be the reference axis. So, phase of the alternating current shown in the Fig. at the instant A, Φ = 0°. While the phase of the current at the instant B is the angle Φ through which the phasor has travelled, measured from the reference axis i.e. X-axis.

In general, the phase Φ of an alternating quantity varies from Φ = 0 to2n radians or Φ=0° to 360°.

In terms of phase the equation of alternating quantity can be modified as,

$$e = E_m \sin(\omega t \pm \phi)$$

Where Φ = Phase of the alternating quantity.
Let us consider three cases;

Case 1 : Φ = 0°

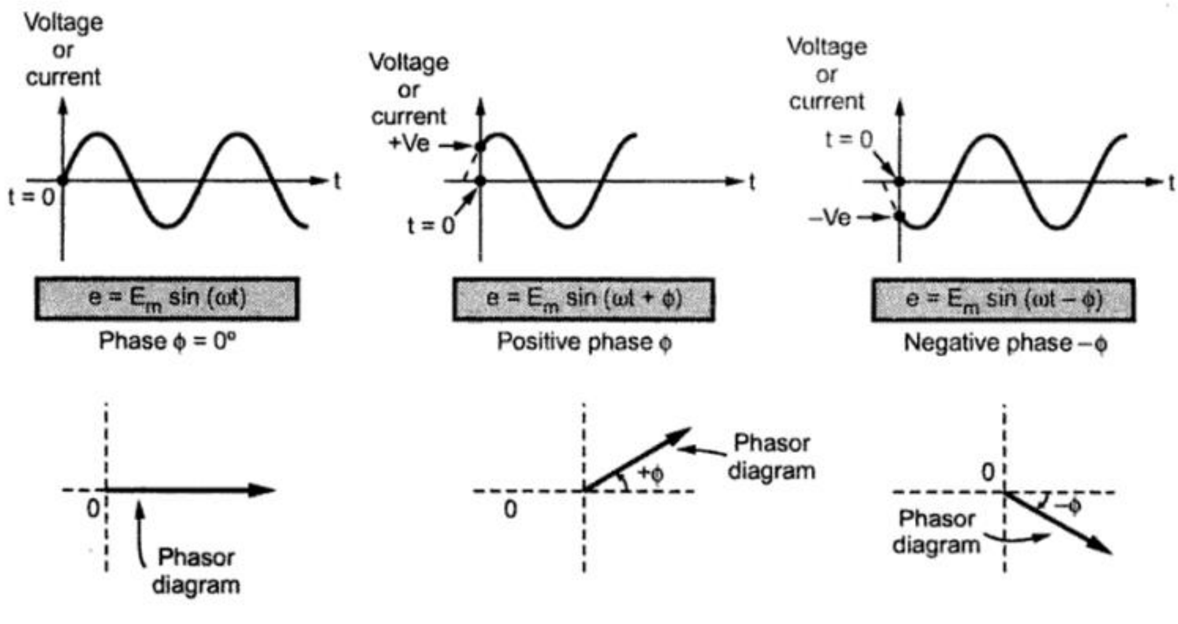
When phase of an alternating quantity is zero, it is standard pure sinusoidal quantity having instantaneous value zero at t = 0. This is shown in the Fig.(a).

Case 2 : Positive phase Φ

When phase of an alternating quantity is positive it means that quantity has some positive instantaneous value at t = 0. This is shown in the Fig.(b).

Case 3 : Negative phase Φ

When phase of an alternating quantity is negative it means that quantity has some negative instantaneous value at t = 0. This is shown in the Fig.(c).



(a) Zero phase **(b) Positive phase** **(c) Negative phase**

Note:

- The phase is measured with respect to reference direction i.e. positive X-axis direction.

AVRK

- The phase measured in anticlockwise direction is positive while the phase measured in clockwise direction is negative.

CONCEPT OF PHASE DIFFERENCE

Consider the two alternating quantities having same frequency f Hz having different maximum values.

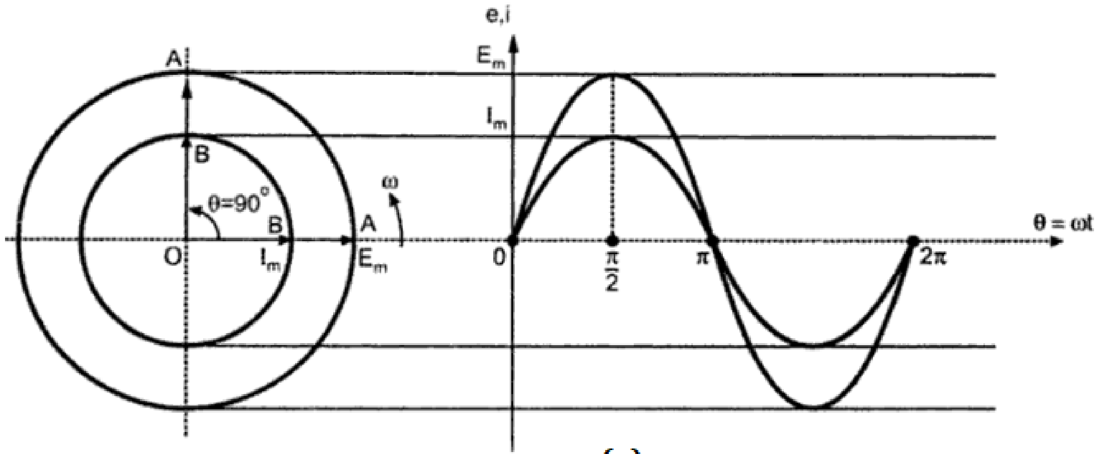
$$e = E_m \sin(\omega t)$$

And $$i = I_m \sin(\omega t)$$

Where $$E_m > I_m$$

The phasor representation and waveforms of both the quantities are shown in the Fig.

The phasors $OA = E_m$ And $OB = I_m$



(a)

After $\theta=\frac{\pi}{2}$ radians, the OA phasor achieves its maximum Em while at the same instant, the OB phasor achieves its maximum I_m. As the frequency of both is same, the angular velocity 'ω' of both is also the same. So, they rotate together in synchronism.

So, at any instant, we can say that the phase of voltage 'e' will be same as phase of 'i'. Thus, the angle travelled by both within a particular time is always the same. So, the difference between the phases of the two quantities is zero at any instant. The difference between the phases of the two alternating quantities is called the **phase difference** which is nothing but the angle difference between the two phasors representing the two alternating quantities.

When such phase difference between the two alternating quantities is zero, the two quantities are said to be in phase.

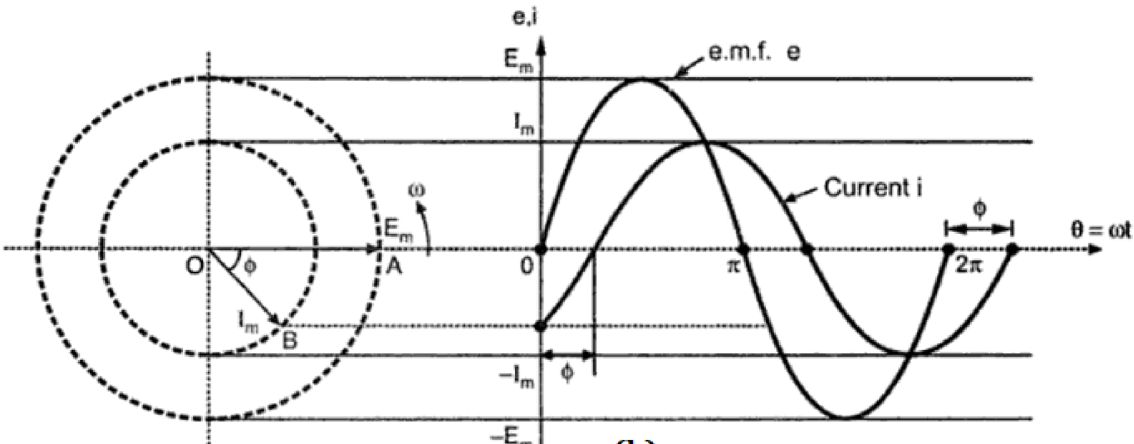
The two alternating quantities having same frequency, reaching maximum positive and negative values and zero values at the same time are said to be in phase. Their amplitudes may be different.

In the a.c. analysis, it is not necessary that all the alternating quantities must be always in phase. It is possible that if one is achieving its zero value, at the same instant, the other is having some negative value or positive value.

Such two quantities are said to have phase difference between them. If there is difference between the phases (angles) of the two quantities, expressed in degrees or radians at any particular instant, then as both rotate with same speed, this difference remains same at all the instants.

Consider an e.m.f. having maximum value Em and current having maximum value Im. Now, when e.m.f. 'e' is at its zero value, the current 'i' has some negative value as shown in the Fig.(b).

Thus, there exists a phase difference Φ between the two phasors. Now, as the two are rotating in anticlockwise direction, we can say that current is falling back with respect to voltage, at all the times by angle Φ. This is called lagging phase difference. The current 'i' is said to lag the voltage 'e' by angle Φ. The current 'i' achieves its maxima, zero values Φ angle later than the corresponding maximum, zero values of voltage.



(b)

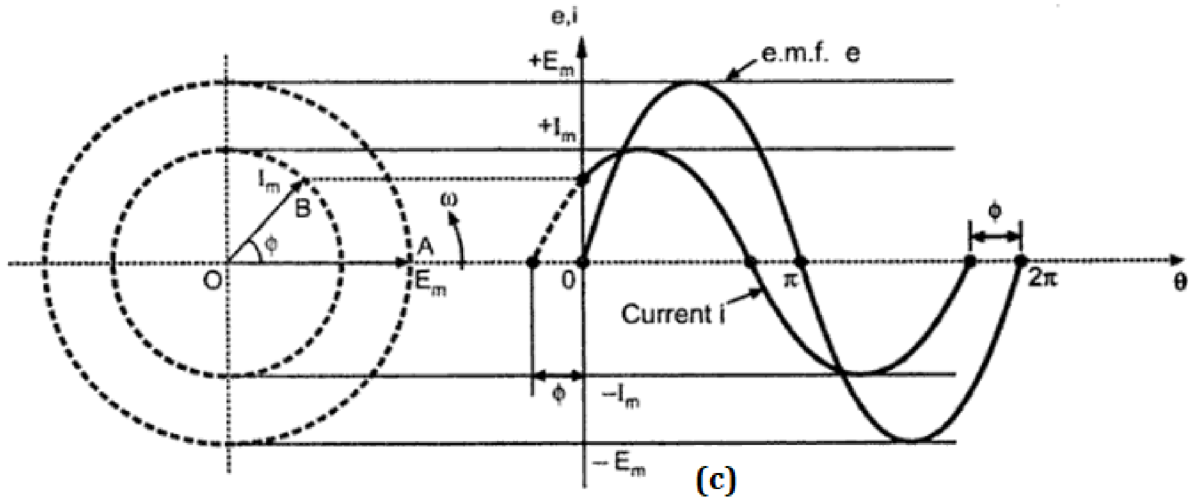
The equations of the two quantities are written as,

$$e = E_m \sin \omega t \quad \text{and} \quad i = I_m \sin(\omega t - \phi)$$

'i' is said to lag 'e' by angle ϕ.

It is possible in practice that the current 'i' may have some positive value when voltage 'e' is zero. This is shown in the Fig.(c).

AVRK



(c)

It can be seen that there exists a phase difference of Φ angle between the two. But in this case, current 'i' is ahead of voltage V, as both are rotating in anticlockwise direction with same speed. Thus, current is said to be leading with respect to voltage and the phase difference is called leading phase difference. The current 'i' achieves its maximum, zero values Φ angle before than the corresponding maximum, zero values of the voltage. At all instants, current i is going to remain ahead of voltage 'e' by angle Φ.
The equations of such two quantities are written as

$e = E_m \sin \omega t$ and $i = I_m \sin (\omega t + \phi)$

'i' is said to lead 'e' by angle ϕ.

Thus, related to the phase difference, it can be remembered that a plus (+) sign of angle indicates lead where as a minus (-) sign of angle indicates lag with respect to the reference.

CONCEPT OF IMPEDANCE, REACTANCE, ADMITTANCE, SUSCEPTANCE

i) IMPEDANCE:(Z)

Electrical impedance is the measure of the opposition that a circuit presents to a current when a voltage is applied.
In Cartesian form, impedance is defined as

$$Z = R + jX$$
$$Z = \sqrt{R^2 + X^2}$$

where the real part of impedance is the resistance, R and the imaginary part is the reactance, X.
For pure Resistor : $Z_R = R$
For pure inductor : $Z_L = j\omega L$, where ω=Angular Frequency = 2Πf
For Pure capacitor: $Z_C = \dfrac{1}{j\omega C} = -\dfrac{j}{\omega C}$

ii) REACTANCE:(X)

Reactance (X) is the imaginary part of the impedance; a component with a finite reactance induces a phase shift between the voltage across it and the current through it. A pure reactance dose not dissipate any power.

$$X = |Z|\sin\theta$$

For pure Resistor : $X_R = 0$
For pure inductor : $X_L = \omega L = 2\Pi f L$
For Pure capacitor: $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi fC}$
Total reactance is $X = X_L + X_C$

iii) ADMITTANCE(Y) & SUSCEPTANCE (B):

In electrical engineering, admittance is a measure of how easily a circuit or device will allow a current to flow.

It is defined as the reciprocal of impedance. The unit is **mho.**

Admittance is defined as

$$Y \equiv \frac{1}{Z}$$

where

Y is the admittance,

Z is the impedance,

$$Y = G + jB$$

where

- Y is the admittance,
- G is the conductance,
- B is the susceptance, measured in siemens.

The impedance, Z, is composed of real and imaginary parts,

$$Z = R + jX$$

$$Y = Z^{-1} = \frac{1}{R + jX} = \left(\frac{1}{R^2 + X^2}\right)(R - jX)$$

$$Y = G + jB$$

where G (conductance) and B (susceptance) are given by:

$$G = \frac{R}{R^2 + X^2}$$

$$B = -\frac{X}{R^2 + X^2}$$

The magnitude of the admittance are given by:

$$|Y| = \sqrt{G^2 + B^2} = \frac{1}{\sqrt{R^2 + X^2}}$$

TYPES OF POWERS

In AC circuits, there are three types of powers. They are

AVRK

i) Real Power ii) Reactive Power iii) Apparent Power

i) REAL POWER/TRUE POWER/ACTIVE POWER/USEFUL POWER (P):

The power which is actually consumed or utilized in an AC Circuit is called True power or Active Power or real power. It is measured in kilo watt (kW) or MW. It is the actual outcomes of the electrical system which runs the electric circuits or load.

It is denoted by (P) and measured in units of Watts (W) i.e. The unit of Active or Real power is Watt.

But the situation in Sinusoidal or AC Circuits is more complex because of phase difference (θ) between Current and Voltage. Therefore average value of power (Real Power) is **P = VI Cosθ** is in fact supplied to the load.

Real Power formulas:

- **P = V I** (In DC circuits)
- **P = VI Cosθ** (in Single phase AC Circuits)
- **$P = \sqrt{3}\, V_L\, I_L\, \cos\theta$** or (in Three Phase AC Circuits)
- **$P = 3\, V_{Ph}\, I_{Ph}\, \cos\theta$**
- **$P = \sqrt{(S^2 - Q^2)}$** or
- **$P = \sqrt{(VA^2 - VAR^2)}$ or**

Real or True power = √ (Apparent Power² – Reactive Power²) or

- **$kW = \sqrt{(kVA^2 - kVAR^2)}$**

ii) REACTIVE POWER/USELESS POWER/WATT LESS POWER (Q):

The powers that continuously bounce back and forth between source and load is known as reactive Power (Q). Power merely absorbed and returned in load due to its reactive properties is referred to as reactive power.

Reactive power represent that the energy is first stored and then released in the form of magnetic field or electrostatic field in case of inductor and capacitor respectively.

Reactive power is given by **Q = V I Sinθ** which can be positive (+ve) for inductive loads and negative (-Ve) for capacitive load.

The unit of reactive power is **Volt-Ampere reactive** i.e. **VAR**

Reactive Power Formulas:

- **Q = V I Sinθ**
- **Reactive Power=√ (Apparent Power² – True power²)**
- **$VAR = \sqrt{(VA^2 - P^2)}$**
- **$kVAR = \sqrt{(kVA^2 - kW^2)}$**

iii) APPARENT POWER/COMPLEX POWER (S):

AVRK

The product of voltage and current if and only if the phase angle differences between current and voltage are ignored. Total power in an AC circuit, both dissipated and absorbed/returned is referred to as apparent power

The combination of reactive power and true power is called apparent power. In an AC circuit, the product of the r.m.s voltage and the r.m.s current is called apparent power which is denoted by (S) and measured in units of Volt-amp (VA). It is the product of Voltage and Current without phase angle.

$$S = V\,I$$

The unit of Apparent power (S) : **VA**

When the circuit is pure resistive, then apparent power is equal to real or true power, but in inductive or capacitive circuit, (when Reactances exist) then apparent power is greater than real or true power.

Apparent Power Formulas:

- $S = V\,I$
- Apparent Power = $\sqrt{(\text{True power}^2 + \text{Reactive Power}^2)}$
- $kVA = \sqrt{kW^2 + kVAR^2}$

Complex Power: $S = P+jQ$ or $S=VI^*$

Complex Power in Capacitive Loads

- $Z = R - jX_C$
- $I = I_P + jI_Q$
- $\cos f = R / |Z|$ (leading)
- $I^* = I_P - jI_Q$
- $S = P - jQ$

Complex Power in Inductive Loads

- $Z = R + jX_L$
- $I = I_P - jI_Q$
- $\cos f = R / |Z|$ (lagging)
- $I^* = I_P + jI_Q$
- $S = P + jQ$

POWER FACTOR (COS Φ)

Power Factor is a measure of how effectively incoming power is used in your electrical system and is defined as the ratio of Real (working) power to Apparent (total) power.

$$\text{Power factor, } \cos\phi = \frac{\text{active power}}{\text{apparent power}} = \frac{kW}{kVA}$$

The power factor of a circuit can be defined in one of the following three ways :

$$\text{Power factor} = \cos\phi = \text{cosine of angle between } V \text{ and } I$$

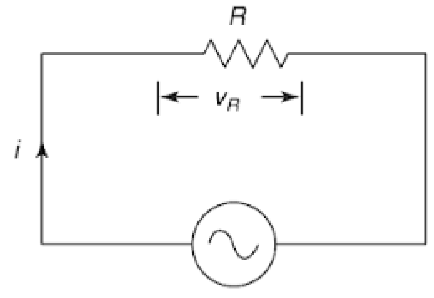
$$\text{Power factor} = \frac{R}{Z} = \frac{\text{Resistance}}{\text{Impedance}}$$

$$\text{Power factor} = \frac{VI\cos\phi}{VI} = \frac{\text{Active power}}{\text{Apparent Power}}$$

BEHAVIOUR OF R, L & C IN AC CIRCUITS

Case-(i) : Behaviour of Pure Resistor in an AC circuit

Consider a pure resistor R connected across an alternating voltage source v as shown in Fig. Let the alternating voltage be $v = V_m \sin \omega t$.

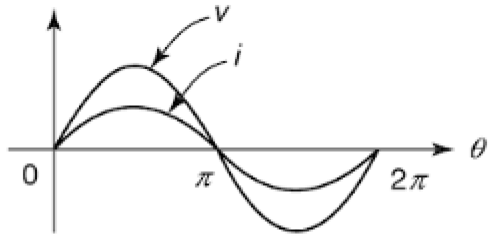


Current The alternating current i is given by

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t \qquad \ldots\left(I_m = \frac{V_m}{R} \right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current is in phase with the voltage in a purely resistive circuit.

Waveforms Figure shows the voltage and current waveforms.



Phasor Diagram Figure shows the phasor diagram.

Impedance It is the resistance offered to the flow of current in an ac circuit. In a purely resistive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{R}} = R$$

Phase Difference Since the voltage and current are in phase with each other, the phase difference is zero.

$$\phi = 0°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{Power factor} = \cos \phi = \cos (0°) = 1$$

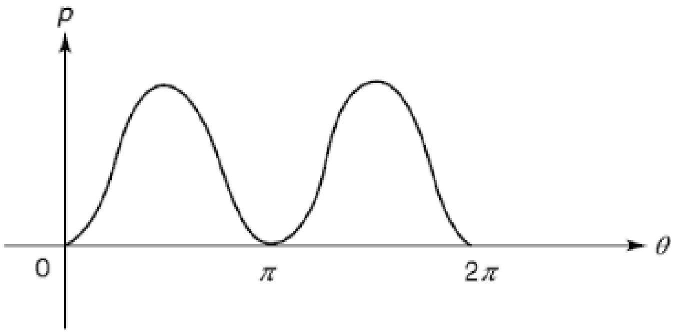
Power Instantaneous power p is given by

$$p = vi$$
$$= V_m \sin \omega t \; I_m \sin \omega t$$
$$= V_m I_m \sin^2 \omega t$$
$$= \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$
$$= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

The power waveform is shown in Fig. · The power consists of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2}$ cos $2\omega t$. The frequency of the fluctuating power is twice the applied voltage frequency and its average value over one complete cycle is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

Thus, power in a purely resistive circuit is equal to the product of rms values of voltage and current.

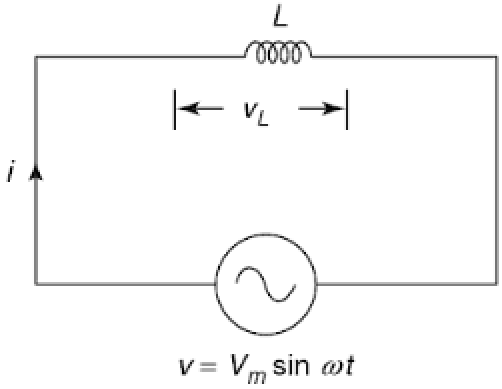


Reactive Power, Q = S sin ϕ = VI sin 0° = 0

"The resistance consumes only active power and the reactive power in a resistance is zero".

Case-(ii) : Behaviour of Pure Inductor in an AC circuit

Consider a pure inductor L connected across an alternating voltage v as shown in Fig. Let the alternating voltage be $v = V_m \sin \omega t$.

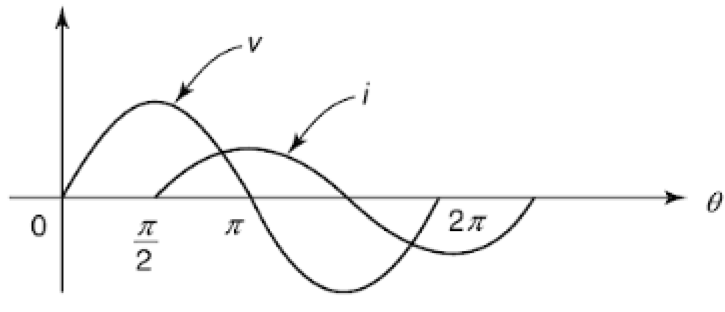


Current The alternating current i is given by

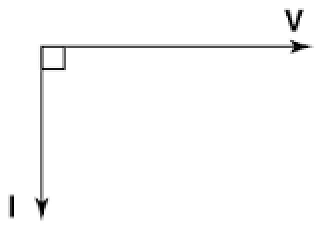
$$i = \frac{1}{L}\int v\, dt$$
$$= \frac{1}{L}\int V_m \sin \omega t\, dt$$
$$= \frac{V_m}{\omega L}(-\cos \omega t)$$
$$= \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$$
$$= I_m \sin\left(\omega t - \frac{\pi}{2}\right) \qquad \ldots\left(I_m = \frac{V_m}{\omega L}\right)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current lags behind the voltage by 90° in a purely inductive circuit.

Waveforms Figure shows the voltage and current waveforms.



Phasor Diagram Figure shows the phasor diagram.



AVRK

Impedance In a purely inductive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{\omega L}} = \omega L$$

The quantity ωL is called inductive reactance, is denoted by X_L and is measured in ohms.
For a dc supply, $f = 0 \quad \therefore \quad X_L = 0$
Thus, an inductor acts as a short circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

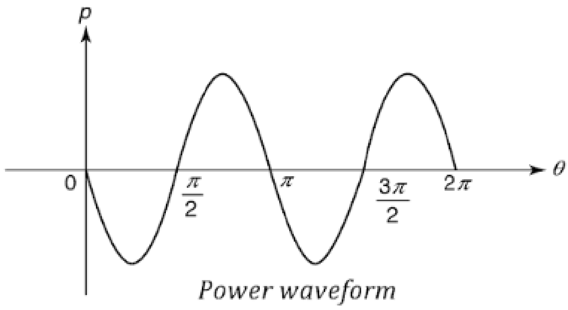
$$\phi = 90°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power Instantaneous powers p is given by

$$\begin{aligned} p &= vi \\ &= V_m \sin\omega t \; I_m \sin\left(\omega t - \frac{\pi}{2}\right) \\ &= -V_m I_m \sin\omega t \cos\omega t \\ &= -\frac{V_m I_m}{2}\sin 2\,\omega t \end{aligned}$$



Power waveform

The power waveform is shown in Fig.
The average power for one complete cycle, $P = 0$.
Hence, power consumed by a purely inductive circuit is zero.

Complex Power, $\bar{S} = \bar{V}\bar{I}^{*} = V\angle 0° \times (I\angle -90°)^{*}$

$$= V\angle 0° \times I\angle +90°$$

$$= VI\angle 90°$$

We know that, $|\bar{S}| = S = VI$ and $\angle \bar{S} = \phi = 90°$

$\therefore$ Power, $P = S\cos\phi = VI\cos 90° = 0$

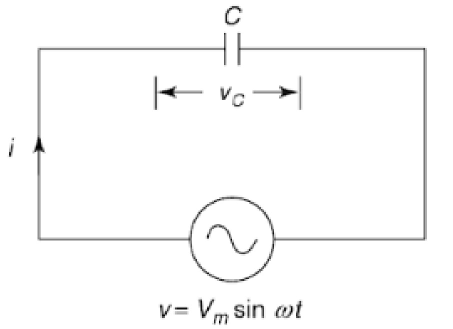
Reactive Power, $Q = S\sin\phi = VI\sin 90° = VI$

The inductance consumes only reactive power and "*the active power in the pure inductance is zero.*" The reactive power of inductance is positive which means that it absorbs reactive power.

Case-(iii) : Behaviour of Pure Capacitor in an AC circuit

Consider a pure capacitor C connected across an alternating voltage v as shown in Fig.
Let the alternating voltage be $v = V_m \sin\omega t$.

AVRK

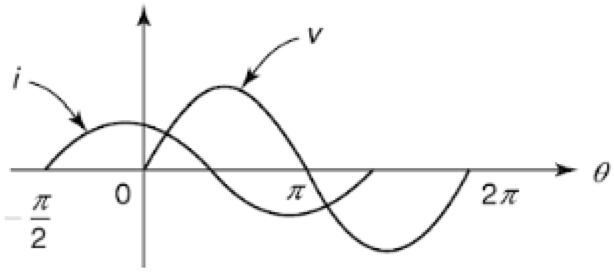


Current The alternating current i is given by

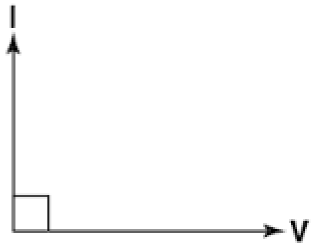
$$i = C\frac{dv}{dt}$$
$$= C\frac{d}{dt}(V_m \sin \omega t)$$
$$= \omega C\, V_m \cos \omega t$$
$$= \omega C\, V_m \sin(\omega t + 90°)$$
$$= I_m \sin(\omega t + 90°) \qquad \ldots(I_m = \omega C\, V_m)$$

where I_m is the maximum value of the alternating current. From the voltage and current equation, it is clear that the current leads the voltage by 90° in a purely capacitive circuit.

Waveforms Figure shows the voltage and current waveforms.



Phasor Diagram Figure shows the phasor diagram.



Impedance In a purely capacitive circuit,

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C\, V_m} = \frac{1}{\omega C}$$

The quantity $\frac{1}{\omega C}$ is called capacitive reactance, is denoted by X_C and is measured in ohms.
For a dc supply, $f = 0 \therefore X_C = \infty$
Thus, the capacitor acts as an open circuit for a dc supply.

Phase Difference It is the angle between the voltage and current phasors.

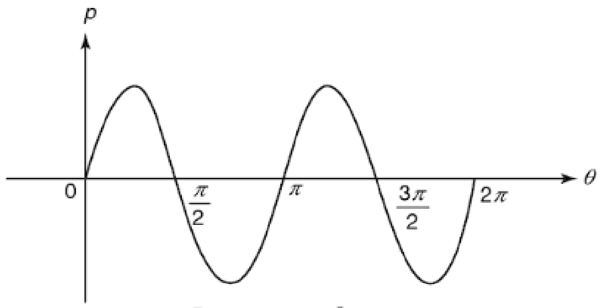
$$\phi = 90°$$

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

Power Instantaneous power p is given by

$$p = vi$$
$$= V_m \sin\omega t\, I_m \sin(\omega t + 90°)$$
$$= V_m I_m \sin\omega t \cos\omega t$$
$$= \frac{V_m I_m}{2} \sin 2\omega t$$



Power waveform

The power waveform is shown in Fig.
The average power for one complete cycle, $P = 0$.
Hence, power consumed by a purely capacitive circuit is zero.

$$\text{Complex Power, } \overline{S} = \overline{V}\,\overline{I}^* = V\angle 0° \times (I\angle 90°)^*$$
$$= V\angle 0° \times I\angle -90°$$
$$= VI\angle -90°$$

We know that, $|\overline{S}| = S = VI$ and $\angle\overline{S} = \phi = -90°$

$$\therefore \text{Power, } P = S\cos\phi = VI\cos(-90°) = 0$$

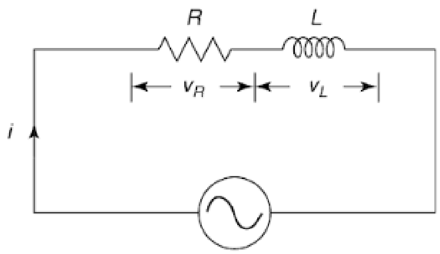
$$\text{Reactive power, } Q = S\sin\phi = VI\sin(-90°) = -VI$$

The capacitance has only reactive power and "*the active power in the pure capacitance is zero*". The reactive power of capacitance is negative which means that it delivers reactive power.

BEHAVIOUR OF SERIES RL CIRCUIT IN AN AC CIRCUIT

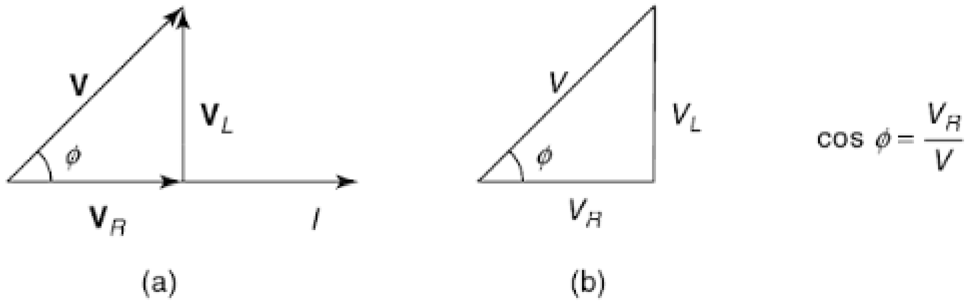
Figure shows a pure resistor R connected in series with a pure inductor L across an alternating voltage $v = V_m \sin\omega t$
Let V and I be the rms values of applied voltage and current.
Potential difference across the resistor $= V_R = RI$
Potential difference across the inductor $= V_L = X_L I$

The voltage $\mathbf{V}_R$ is in phase with the current $\mathbf{I}$ whereas the voltage $\mathbf{V}_L$ leads the current $\mathbf{I}$ by 90°.



Phase Diagram

Figure shows the phasor diagram of series RL circuit.



(a) Phasor Diagram, (b) Voltage triangle

Impedance

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_L = R\mathbf{I} + jX_L\mathbf{I} = (R + jX_L)\,\mathbf{I}$$

$$\frac{\mathbf{V}}{\mathbf{I}} = R + jX_L = \mathbf{Z}$$

$$\mathbf{Z} = Z\angle\phi$$

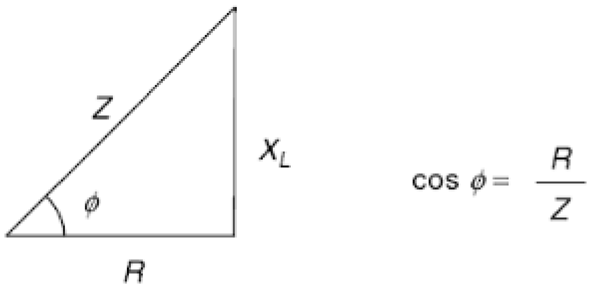
$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The quantity Z is called the *complex impedance* of the series RL circuit.

Impedance Triangle

Figure shows the impedance triangle of series RL circuit.



Current From the phasor diagram, it is clear that the current I lags behind the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin\omega t$ then the current equation will be

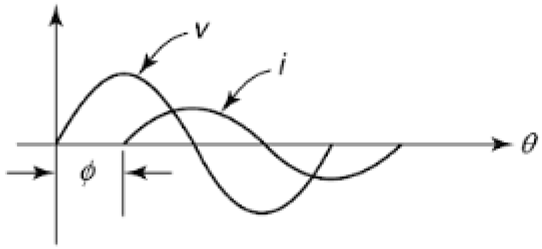
$$i = I_m \sin(\omega t - \phi)$$

where $$I_m = \frac{V_m}{Z}$$

and $$\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

AVRK

Waveforms Figure shows the voltage and current waveforms.



Power Instantaneous power p is given by

$$p = vi$$
$$= V_m \sin \omega t \, I_m \sin (\omega t - \phi)$$
$$= V_m I_m \sin \omega t \sin (\omega t - \phi)$$
$$= V_m I_m \left[\frac{\cos \phi - \cos (2\omega t - \phi)}{20} \right]$$
$$= \frac{V_m I_m}{2} \cos \phi - \frac{V_m I_m}{2} \cos (2\omega t - \phi)$$

Thus, power consists of a constant part $\frac{V_m I_m}{2} \cos \phi$ and a fluctuating part $\frac{V_m I_m}{2} \cos (2\omega t - \phi)$. The frequency of the fluctuating part is twice the applied voltage frequency and its average value over one complete cyce is zero.

$$\text{Average power } P = \frac{V_m I_m}{2} \cos \phi = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos \phi = VI \cos \phi$$

Thus, power is dependent upon the in-phase component of the current. The average power is also called *active power* and is measured in watts.

We know that a pure inductor and capacitor consume no power because all the power received from the source in a half cycle is returned to the source in the next half cycle. This circulating power is called *reactive power*. It is a product of the voltage and reactive component of the current, i.e., $I \sin \phi$ and is measured in VAR (volt-ampere-reactive).

Reactive power $Q = VI \sin \phi$.

The product of voltage and current is known as *apparent power* (S) and is measured in volt-ampere (VA).

$$S = VI = \sqrt{P^2 + Q^2}$$

Power Triangle In terms of circuit components,

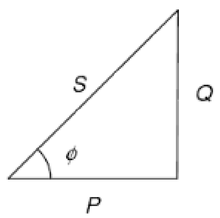
$$\cos \phi = \frac{R}{Z}$$

and $$V = ZI$$

$$P = VI \cos \phi = ZI I \frac{R}{Z} = I^2 R$$

$$Q = VI \sin \phi = ZII \frac{X_L}{Z} = I^2 X_L$$

$$S = VI = Z I I = I^2 Z$$



$$\cos \phi = \frac{P}{S}$$

Figure shows the power triangle of series RL circuit.

AVRK

Power Factor It is defined as the cosine of the angle between the voltage and current phasors.

$$\text{pf} = \cos\phi$$

From voltage triangle, $\text{pf} = \dfrac{V_R}{V}$

From impedance triangle, $\text{pf} = \dfrac{R}{Z}$

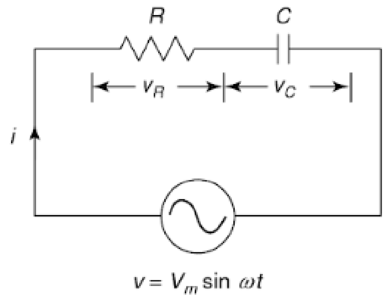
From power triangle, $\text{pf} = \dfrac{P}{S}$

In case of series *RL* circuit, the power factor is lagging in nature.

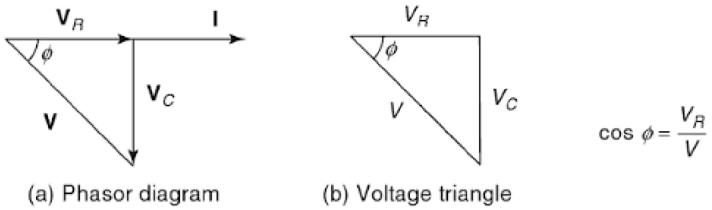
BEHAVIOUR OF SERIES RC CIRCUIT IN AN AC CIRCUIT

Figure shows a pure resistor *R* connected in series with a pure capacitor *C* across an alternating voltage $v = V_m \sin \omega t$.
Let *V* and *I* be the rms values of applied voltage and current.
Potential difference across the resistor = $V_R = R\,I$
Potential difference across the capacitor = $V_C = X_C I$
The voltage $\mathbf{V}_R$ is in phase with the current $\mathbf{I}$ whereas voltage $\mathbf{V}_c$ lags behind the current $\mathbf{I}$ by 90°.

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_C$$



Phasor Diagram Since the same current flows through *R* and *C*, the current *I* is taken as a reference phasor. Figure shows the phasor diagram of series *RC* circuit.



(a) Phasor diagram (b) Voltage triangle

Impedance

$$\mathbf{V} = \mathbf{V}_R + \mathbf{V}_C = R\mathbf{I} - jX_C\mathbf{I} = (R - jX_C)\mathbf{I}$$

$$\frac{\mathbf{V}}{\mathbf{I}} = R - jX_C = \mathbf{Z}$$

$$\mathbf{Z} = Z\angle -\phi$$

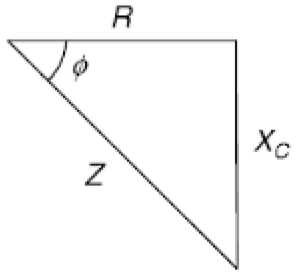
$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

The quantity $\mathbf{Z}$ is called the *complex impedance* of the series *RC* circuit.

Impedance Triangle

Figure shows the impedance triangle of series *RC* circuit.



$$\cos \phi = \frac{R}{Z}$$

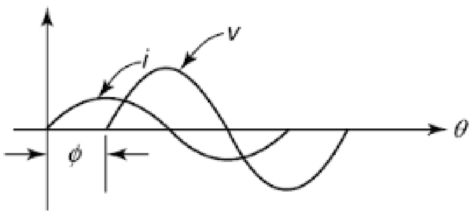
Current From the phasor diagram, it is clear that the current I leads the voltage V by an angle ϕ. If the applied voltage is given by $v = V_m \sin \omega t$ then the current equation will be

$$i = I_m \sin(\omega t + \phi)$$

where $I_m = \dfrac{V_m}{Z}$

and $\phi = \tan^{-1}\left(\dfrac{X_C}{R}\right) = \tan^{-1}\left(\dfrac{1}{\omega RC}\right)$

Waveforms Figure shows the voltage and current waveforms.



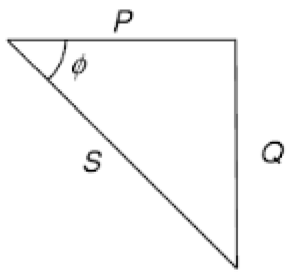
Power

Active power $P = VI\cos\phi = I^2R$

Reactive power $Q = VI\sin\phi = I^2X_C$

Apparent power $S = VI = I^2Z$

Power Triangle Figure shows the power triangle of series *RC* circuit



$$\cos \phi = \frac{P}{S}$$

AVRK

Power Factor It is defined as the cosine of the angle between voltage and current phasors.

$$pf = \cos\phi$$

From voltage triangle, $pf = \frac{V_R}{V}$

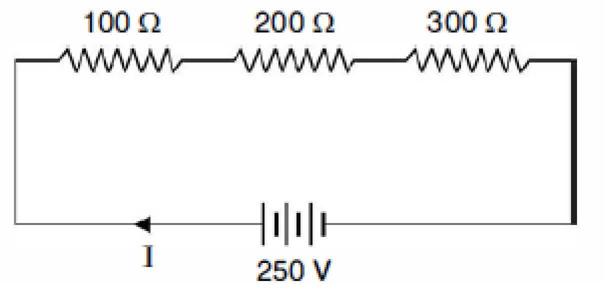
From impedance triangle $pf = \frac{R}{Z}$

From power triangle, $pf = \frac{P}{S}$

In case of series RC circuit, the power factor is leading in nature.

SOLVED PROBLEMS

1) Three resistances 100 Ω, 200 Ω and 300 Ω are connected in series to a 250 volt supply. Determine the current in the circuit and the power dissipated in each resistor.



SOL:

The total resistance in the circuit

$$R_T = R_1 + R_2 + R_3 = 100 + 200 + 300 = 600\ \Omega$$

$$\text{Current } I = \frac{V}{R_T} = \frac{250}{600} = 0.417 \text{ Amps.}$$

Power loss in 100 Ω resistor = $I^2R_1 = 0.417^2 \times 100 = 17.36$ watts.

Power loss in 200 Ω resistor = $I^2R_2 = 0.417^2 \times 200 = 34.72$ watts.

Power loss in 300 Ω resistor = $I^2R_3 = 0.417^2 \times 300 = 52$ watts.

Total power loss = 104 watts.

2) When a dc voltage is applied to a capacitor, the voltage across its terminals is found to build up in accordance with $V_C = 50(1 - e^{-100t})$. After a lapse of 0.01s, the current flow is equal to 2 mA.

(a) Find the value of capacitance in microfarads.

(b) How much energy is stored in the electric field at this time?

SOL:

AVRK

(a) $i = C\dfrac{dv_C}{dt}$

where $v_C = 50(1 - e^{-100t})$

$$i = C\frac{d}{dt}50\left(1 - e^{-100t}\right)$$

$$= C \times 50 \times 100e^{-100t}$$

At $t = 0.01$ s, $i = 2$ mA

$$C = \frac{2\times10^{-3}}{50\times100\times e^{-100\times0.01}} = 1.089\ \mu F$$

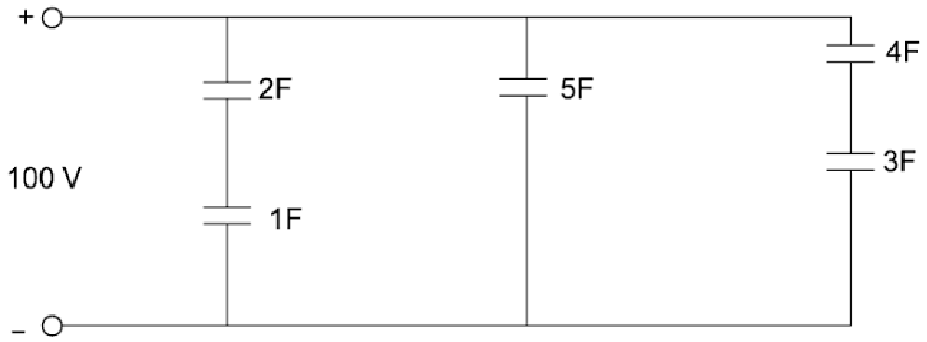
(b) $W = \dfrac{1}{2}Cv_C^2$

At $t = 0.01$ s, $v_C = 50\,(1 - e^{-100\times0.01}) = 31.6$ V

$$W = \frac{1}{2}\times1.089\times10^{-6}\times(31.6)^2$$

$$= 0.000543 \text{ J}$$

3) Find the total equivalent capacitance and total energy stored if the applied voltage is 100V for the circuit shown in the fig.



SOL:

AVRK

4F & 3F in series

$$C_{eq} = \frac{4\times3}{4+3} = \frac{12}{4}\ F$$

$\frac{12}{7}$ F in parallel with 5 F

$$\therefore \quad C_{eq} = \frac{12}{7} + 5 = \frac{35+12}{7} = \frac{47}{7}\ F$$

$\therefore \quad C_{eq}$ = 2 F & 1 F in series

$$C_{eq} = \frac{2\times1}{2+1} = \frac{2}{3} F$$

$\frac{2}{3}$ F in parallel with $\frac{47}{7}$ F

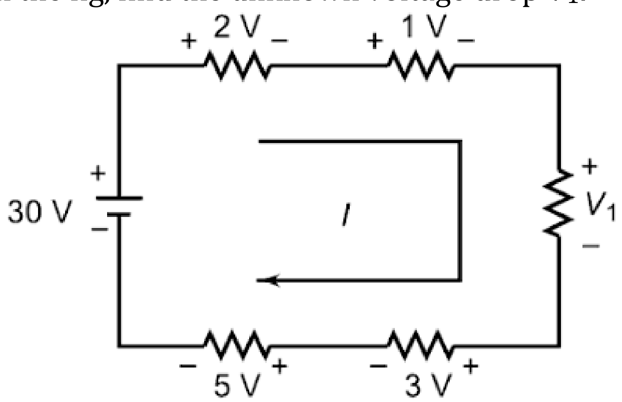
$$\therefore \quad C_{eq} = \frac{2}{3} + \frac{47}{7} = \frac{14+141}{21} = \frac{155}{21} F$$

$$\therefore \quad E = \frac{1}{2}CV^2$$

$$\therefore \quad E = \frac{1}{2}\times\frac{155}{21}\times 100\times 100$$

$$E = 36900\ J$$

4) For the circuit shown in the fig, find the unknown voltage drop V_1.



SOL:

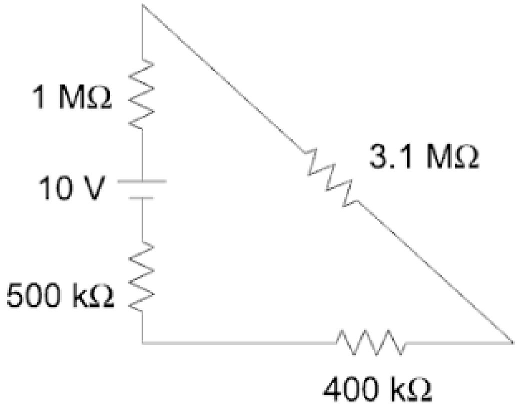
According to Kirchhoff's voltage law, the sum of the potential drops is equal to the sum of the potential rises;

AVRK

Therefore, $30 = 2 + 1 + V_1 + 3 + 5$

or $V_1 = 30 - 11 = 19\text{ V}$

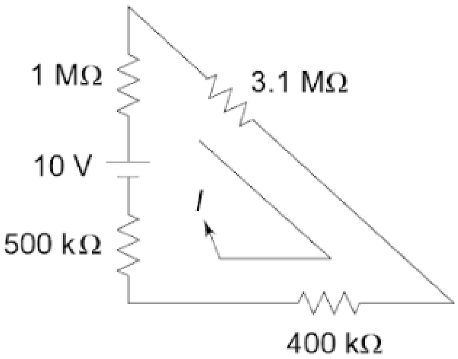
5) Determine the current in the given circuit as shown in the figure by using Kirchhoff's voltage law. Also find the voltage across each resistor.



SOL:

We assume current I in the clockwise direction and indicate polarities

By using Ohm's law, we find the voltage drops across each resistor.



$$V_{1M} = I, \quad V_{3.1M} = 3.1\, I$$

$$V_{500K} = 0.5\, I, \quad V_{400K} = 0.4\, I$$

Now, by applying Kirchhoff's voltage law, we form the equation.

$$10 = I + 3.1\, I + 0.5\, I + 0.4\, I$$

or $5\, I = 10$

or $I = 2\ \mu\text{A}$

∴ Voltage across each resistor is as follows

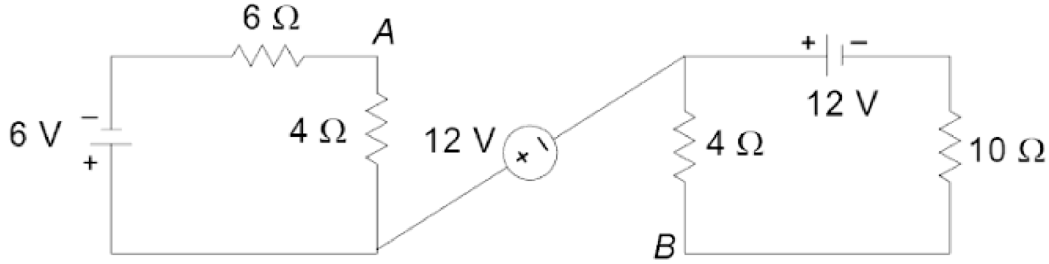
$$V_{1M} = 1 \times 2 = 2.0\text{ V}$$

$$V_{3.1M} = 3.1 \times 2 = 6.2\text{ V}$$

$$V_{400\text{ K}} = 0.4 \times 2 = 0.8\text{ V}$$

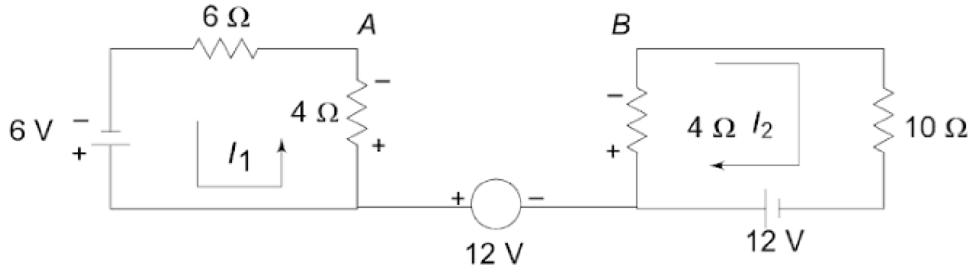
$$V_{500\text{ K}} = 0.5 \times 2 = 1.0\text{ V}$$

6) Determine the voltage across A and B in the circuit given below.



SOL:

The above circuit can be redrawn as shown in Fig. Assume loop currents I_1 and I_2 as shown in Fig.

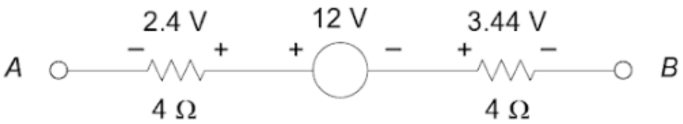


$$I_1 = \frac{6}{10} = 0.6\text{ A}$$

$$I_2 = \frac{12}{14} = 0.86\text{ A}$$

V_A = Voltage drop across 4 Ω resistor = 0.6 × 4 = 2.4 V

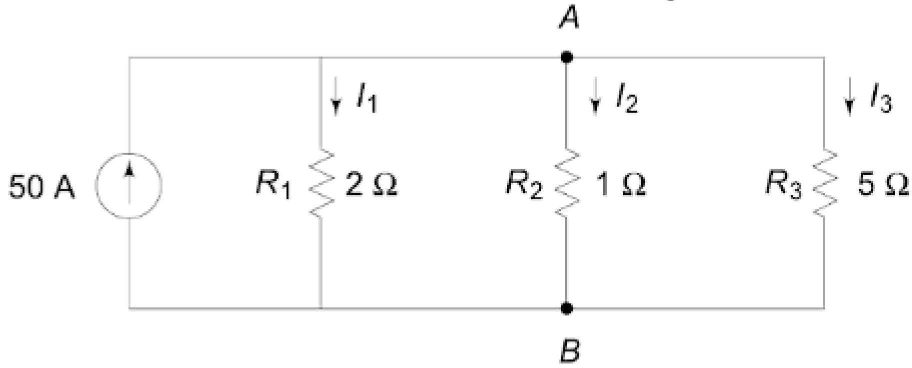
V_B = Voltage drop across 4 Ω resistor = 0.86 × 4 = 3.44 V



The voltage between points A and B is the sum of voltages as shown in Fig.

$$\therefore \quad V_{AB} = -2.4 + 12 + 3.44 = 13.04\text{ V}$$

7) Determine the current in all resistors in the below circuit using Kirchhoff's current law.



AVRK

SOL:

The above circuit contains a single node 'A' with reference node 'B'. Our first step is to assume the voltage V at node A. In a parallel circuit the same voltage is applied across each element. According to Ohm's law, the currents passing through each element are $I_1 = V/2$, $I_2 = V/1$, $I_3 = V/5$.

By applying Kirchhoff's current law, we have

$$I = I_1 + I_2 + I_3$$

$$I = \frac{V}{2} + \frac{V}{1} + \frac{V}{5}$$

$$50 = V\left[\frac{1}{2} + \frac{1}{1} + \frac{1}{5}\right] = V\,[0.5 + 1 + 0.2]$$

$$V = \frac{50}{1.7} = \frac{500}{17} = 29.41\text{ V}$$

Once we know the voltage V at node A, we can find the current in any element by using Ohm's law.

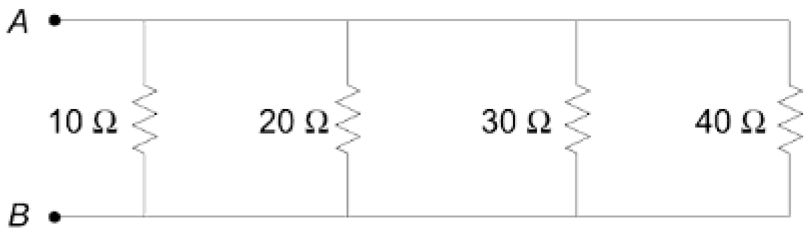
The current in the 2 Ω resistor is $I_1 = 29.41/2 = 14.705$ A.

Similarly $$I_2 = \frac{V}{R_2} = \frac{V}{1} = 29.41\text{ A}$$

$$I_3 = \frac{29.41}{5} = 5.882\text{ A}$$

∴ $I_1 = 14.7$ A, $I_2 = 29.4$ A, and $I_3 = 5.88$ A

8) Determine the parallel resistance between the points A & B of the circuit shown in the fig.



SOL:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

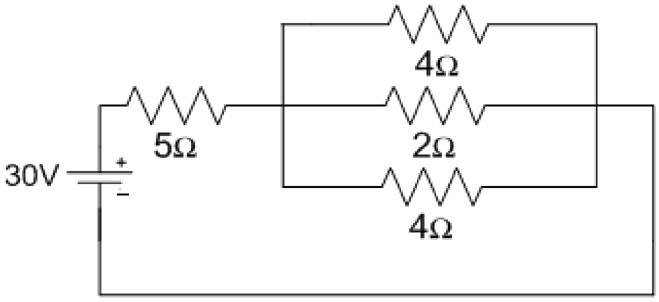
$$\frac{1}{R_T} = \frac{1}{10} + \frac{1}{20} + \frac{1}{30} + \frac{1}{40}$$

$$= 0.1 + 0.05 + 0.033 + 0.025 = 0.208$$

$$R_T = 4.8\ \Omega$$

AVRK

9) Determine the total current in the circuit given below:



SOL:

Resistances R_2, R_3 and R_4 are in parallel

$\therefore$ Equivalent resistance $R_5 = R_2 \parallel R_3 \parallel R_4$

$$= \frac{1}{1/R_2 + 1/R_3 + 1/R_4}$$

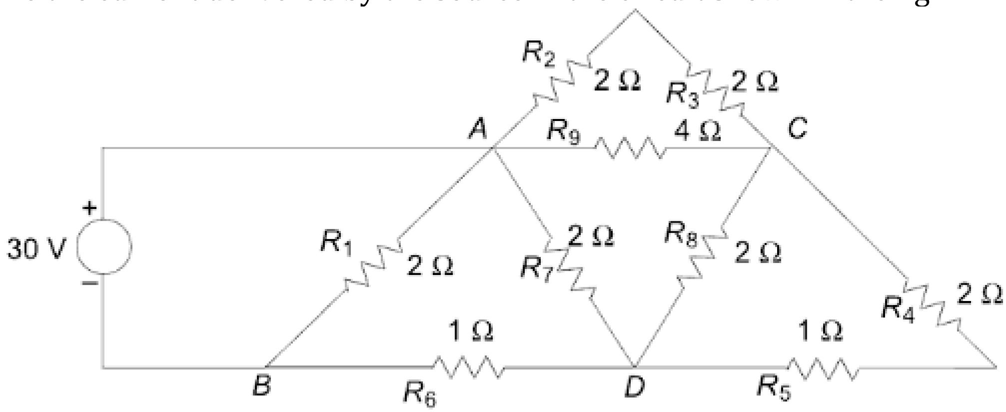
$\therefore \quad R_5 = 1\ \Omega$

R_1 and R_5 are in series,

$\therefore$ Equivalent resistance $R_T = R_1 + R_5 = 5 + 1 = 6\ \Omega$

And the total current $I_T = \dfrac{V_s}{R_T} = \dfrac{30}{6} = 5\ \text{A}$

10) Determine the current delivered by the source in the circuit shown in the fig.

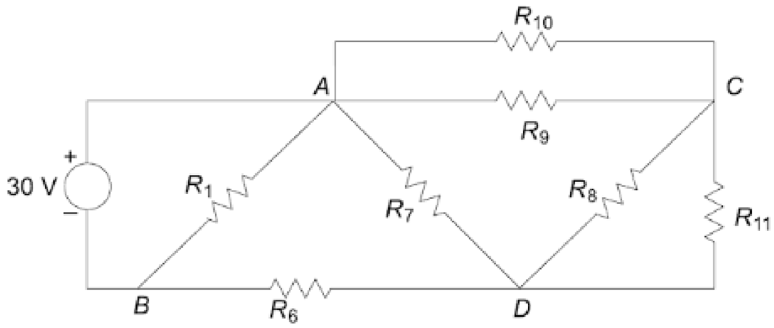


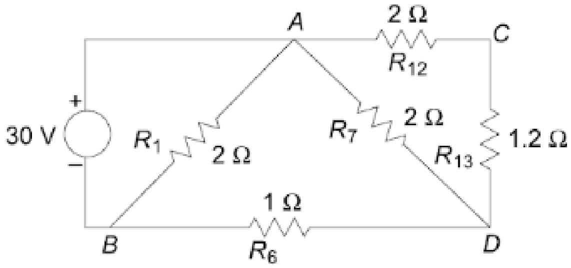
SOL:

The circuit can be modified as shown in Fig. , where R_{10} is the series combination of R_2 and R_3.

$\therefore \quad R_{10} = R_2 + R_3 = 4\ \Omega$

AVRK





R_{11} is the series combination of R_4 and R_5

$\therefore \quad R_{11} = R_4 + R_5 = 3\ \Omega$

Further simplification of the circuit leads to Fig. where R_{12} is the parallel combination of R_{10} and R_9.

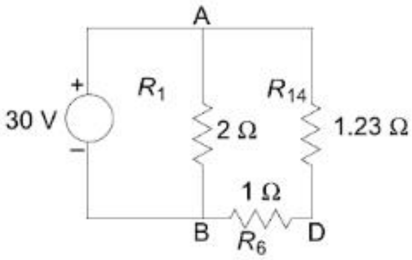
$\therefore \quad R_{12} = (R_{10} \parallel R_9) = (4 \parallel 4) = 2\ \Omega$

Similarly, R_{13} is the parallel combination of R_{11} and R_8

$\therefore \quad R_{13} = (R_{11} \parallel R_8) = (3 \parallel 2) = 1.2\ \Omega$

In Fig. 2.32 as shown, R_{12} and R_{13} are in series, which is in parallel with R_7 forming R_{14}. This is shown in Fig.

$\therefore \quad R_{14} = [(R_{12} + R_{13})//R_7]$



$= [(2 + 1.2)//2] = 1.23\ \Omega$

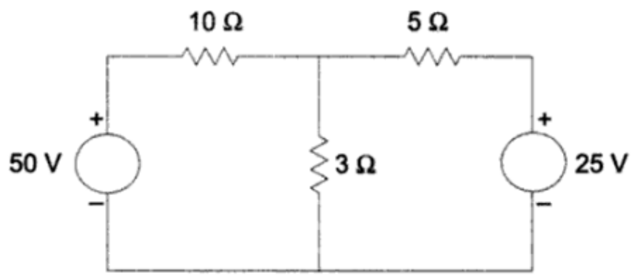
Further, the resistances R_{14} and R_6 are in series, which is in parallel with R_1 and gives the total resistance

$R_T = [(R_{14} + R_6)//R_1]$

$= [(1 + 1.23)//(2)] = 1.05\ \Omega$

The current delivered by the source = 30/1.05 = 28.57 A

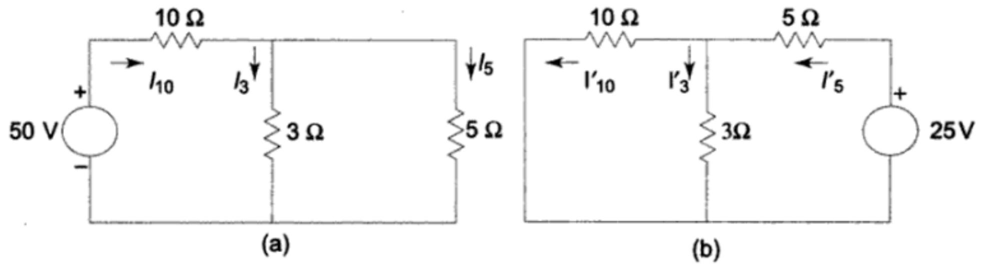
11) For the resistive network shown in the fig, find the current in each resistor using superposition theorem.



SOL:

AVRK

The current due to the 50 V source can be found in the circuit shown in Fig. (a).



$$\text{Total resistance } R_T = 10 + \frac{5 \times 3}{8} = 11.9\ \Omega$$

$$\text{Current in the } 10\ \Omega \text{ resistor } I_{10} = \frac{50}{11.9} = 4.2\ \text{A}$$

$$\text{Current in the } 3\ \Omega \text{ resistor } I_3 = 4.2 \times \frac{5}{8} = 2.63\ \text{A}$$

$$\text{Current in the } 5\ \Omega \text{ resistor } I_5 = 4.2 \times \frac{3}{8} = 1.58\ \text{A}$$

The current due to the 25 V source can be found from the circuit shown in Fig (b).

$$\text{Total resistance } R_T = 5 + \frac{10 \times 3}{13} = 7.31\ \Omega$$

$$\text{Current in the } 5\ \Omega \text{ resistor } I'_5 = \frac{25}{7.31} = 3.42\ \text{A}$$

$$\text{Current in the } 3\ \Omega \text{ resistor } I'_3 = 3.42 \times \frac{10}{13} = 2.63\ \text{A}$$

$$\text{Current in the } 10\ \Omega \text{ resistor } I'_{10} = 3.42 \times \frac{3}{13} = 0.79\ \text{A}$$

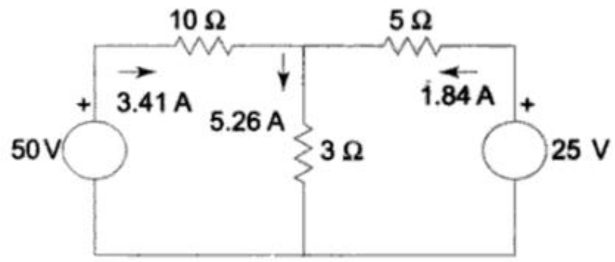
According to superposition principle

Current in the 10 Ω resistor $= I_{10} - I'_{10} = 4.2 - 0.79 = 3.41$ A

Current in the 3 Ω resistor $= I_3 + I'_3 = 2.63 + 2.63 = 5.26$ A

Current in the 5 Ω resistor $= I'_5 - I_5 = 3.42 - 1.58 = 1.84$ A

When both sources are operative, the directions of the currents are shown in Fig. (c).



AVRK

12) An alternating current is given by **i=14.14 sin(377 t).** Find
(a) rms value of the current (b) frequency (c) instantaneous value of the current at t=3 ms
(d) time taken by the current to reach 10A for first time after passing through zero.
SOL:

$$i = 14.14 \sin 377\, t$$

(a) The rms value of the current

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{14.14}{\sqrt{2}} = 10 \text{ A}$$

(b) Frequency

$$2\pi f = 377$$

$$f = \frac{377}{2\pi} = 60 \text{ Hz}$$

(c) Instantaneous value of the current when $t = 3$ ms

$$i = 14.14 \sin(377 \times 3 \times 10^{-3}) \quad \text{(angle in radians)}$$
$$= 12.79 \text{ A}$$

(d) Time taken by the current to reach 10 A for the first time after passing through zero

$$i = 14.14 \sin 377\, t \quad \text{(angle in radians)}$$
$$10 = 14.14 \sin 377\, t$$
$$t = 2.084 \text{ ms}$$

NOTE:

$$\text{Radians} = \left(\frac{\pi}{180°}\right) \times (\text{degrees})$$

$$\text{Degrees} = \left(\frac{180°}{\pi}\right) \times (\text{radians})$$

13) Determine the following parameters of a voltage v = 200 sin314t.
(a) Frequency (b) Form factor (c) Crest factor
SOL:

$$v = 200 \sin 314\, t$$

(a) Frequency

$$v = V_m \sin 2\pi f t$$
$$2\pi f = 314$$
$$f = \frac{314}{2\pi} = 50 \text{ Hz}$$

For a sinusoidal waveform,

$$V_{avg} = \frac{2V_m}{\pi}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

AVRK

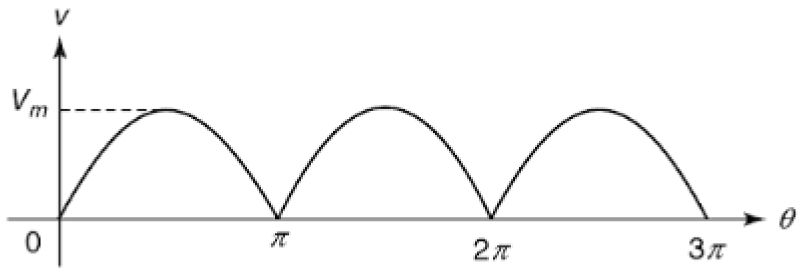
(b) Form factor

$$k_f = \frac{V_{rms}}{V_{avg}} = \frac{\frac{V_m}{\sqrt{2}}}{\frac{2V_m}{\pi}} = 1.11$$

(c) Crest factor

$$k_p = \frac{V_m}{V_{rms}} = \frac{V_m}{\frac{V_m}{\sqrt{2}}} = 1.414$$

14) Determine the average value and rms value of the waveform shown in the fig.



SOL:

$$v = V_m \sin\theta \qquad 0 < \theta < \pi$$

(a) Average value of the waveform

$$V_{avg} = \frac{1}{\pi}\int_0^{\pi} v(\theta)\,d\theta = \frac{1}{\pi}\int_0^{\pi} V_m \sin\theta\,d\theta = \frac{V_m}{\pi}\left[-\cos\theta\right]_0^{\pi}$$

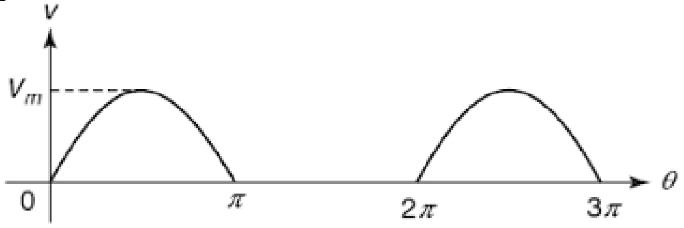
$$= \frac{V_m}{\pi}[1+1] = 0.637\ V_m$$

(b) rms value of the waveform

$$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2(\theta)\,d\theta} = \sqrt{\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\,d\theta} = \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\sin^2\theta\,d\theta}$$

$$= \sqrt{\frac{V_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta} = \sqrt{\frac{V_m^2}{\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}} = \sqrt{\frac{V_m^2}{\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]} = 0.707\ V_m$$

15) Determine the average value and rms value of the waveform shown in the fig.



SOL:

AVRK

$$v = V_m \sin\theta \qquad 0 < \theta < \pi$$
$$= 0 \qquad \pi < \theta < 2\pi$$

(a) Average value of the waveform

$$V_{avg} = \frac{1}{2\pi}\int_0^{2\pi} v(\theta)\, d\theta = \frac{1}{2\pi}\left[\int_0^{\pi} V_m \sin\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m \sin\theta\, d\theta = \frac{V_m}{2\pi}\left[-\cos\theta\right]_0^{\pi} = \frac{V_m}{2\pi}[1+1] = 0.318\, V_m$$

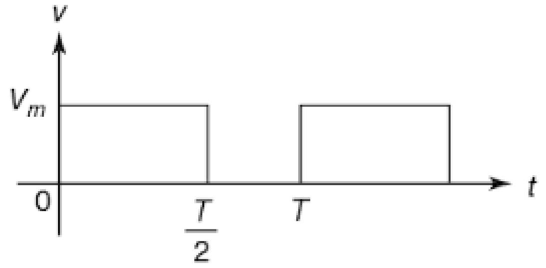
(b) rms value of the waveform

$$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2(\theta)\, d\theta} = \sqrt{\frac{1}{2\pi}\left[\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta + \int_{\pi}^{2\pi} 0\, d\theta\right]}$$

$$= \sqrt{\frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi} \sin^2\theta\, d\theta} = \sqrt{\frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta}$$

$$= \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}} = \sqrt{\frac{V_m^2}{2\pi}\left[\frac{\pi}{2} - \frac{\sin 2\pi}{4} - 0 + \frac{\sin 0}{4}\right]} = 0.5\ V_m$$

16) Determine the average value and rms value of the waveform shown in the fig.



SOL:

$$v = V_m \qquad 0 < t < \frac{T}{2}$$

$$= 0 \qquad \frac{T}{2} < t < T$$

(a) Average value of the waveform

$$V_{avg} = \frac{1}{T}\int_0^{T} v(t)\, dt = \frac{1}{T}\left[\int_0^{\frac{T}{2}} V_m\, dt + \int_{\frac{T}{2}}^{T} 0\, dt\right] = \frac{1}{T}\int_0^{\frac{T}{2}} V_m\, dt$$

$$= \frac{V_m}{T}[t]_0^{\frac{T}{2}} = \frac{V_m}{T}\cdot\frac{T}{2} = 0.5\ V_m$$

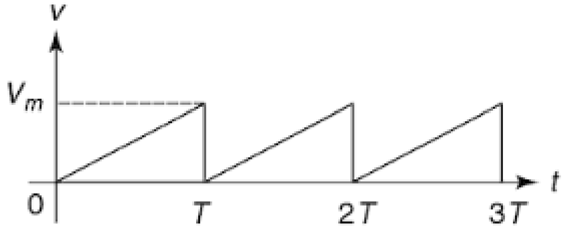
AVRK

(b) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt} = \sqrt{\frac{1}{T}\int_0^{\frac{T}{2}} V_m^2\, dt} = \sqrt{\frac{V_m^2}{T}[t]_0^{\frac{T}{2}}}$$

$$= \sqrt{\frac{V_m^2}{T}\cdot\frac{T}{2}} = 0.707\, V_m$$

17) Determine the average value and rms value of the waveform shown in the fig.



SOL:

$$v = \frac{V_m}{T}t \qquad\qquad 0 < t < T$$

(a) Average value of the waveform

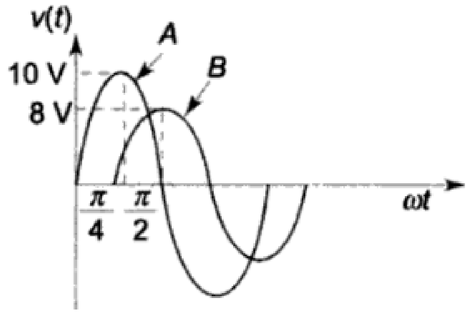
$$V_{\text{avg}} = \frac{1}{T}\int_0^T v(t)\, dt = \frac{1}{T}\int_0^T \frac{V_m}{T} t\, dt = \frac{V_m}{T^2}\left[\frac{t^2}{2}\right]_0^T = \frac{V_m}{T^2}\cdot\frac{T^2}{2} = 0.5\, V_m$$

(b) rms value of the waveform

$$V_{\text{rms}} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\, dt} = \sqrt{\frac{1}{T}\int_0^T \frac{V_m^2}{T^2}\cdot t^2\, dt}$$

$$= \sqrt{\frac{V_m^2}{T^3}\left[\frac{t^3}{3}\right]_0^T} = \sqrt{\frac{V_m^2}{T^3}\left[\frac{T^3}{3}\right]} = 0.577\, V_m$$

18) Determine the instantaneous value at 90^0 point on the X-axis for each sine wave shown in figure



AVRK

SOL:

From Fig. the equation for the sine wave *A*

$$v(t) = 10 \sin \omega t$$

The value at $\pi/2$ in this wave is

$$v(t) = 10 \sin \frac{\pi}{2} = 10 \text{ V}$$

The equation for the sine wave *B*

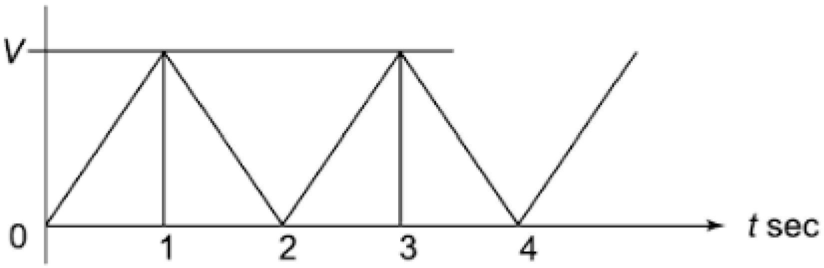
$$v(t) = 8 \sin (\omega t - \pi/4)$$

At $\omega t = \pi/2$

$$v(t) = 8 \sin \left(\frac{\pi}{2} - \frac{\pi}{4}\right)$$

$$= 8 \sin 45° = 8\ (0.707) = 5.66 \text{ V}$$

19) Calculate the Form factor for the following waveform



SOL:

$$\text{Form factor} = \frac{\text{R.M.S. value}}{\text{Average value}}$$

Average value of the triangular waveform 0 to 2 sec

$$V_{av} = \frac{1}{2}\left[\int_0^1 V.t\ dt + \int_1^2 -V(t-2)\,dt\right]$$

$$= \frac{1}{2}\left[V\frac{t^2}{2}\bigg|_0^1 + -V\frac{t^2}{2}\bigg|_1^2 + 2V.t\big|_1^2\right]$$

$$= \frac{1}{2}\left[\frac{V}{2} - \frac{3}{2}V + 2V\right] = \frac{V}{2}$$

$$\text{R.M.S. value, } (V_{rms}) = \left[\frac{1}{2}\int_0^1 V^2t^2dt + \int_1^2 V^2(t-2)^2\,dt\right]^{1/2}$$

AVRK

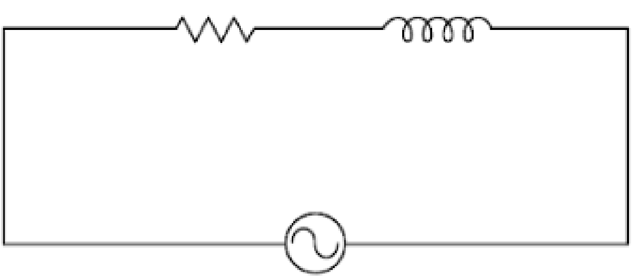
$$= \left[\frac{1}{2}\left\{V^2 \frac{t^3}{3}\Big|_0^1 + V^2 \frac{t^3}{3}\Big| + 4V^2 t\Big|_1^2 - 4V^2 \frac{t^2}{2}\Big|_1^2\right\}\right]^{1/2}$$

$$= \left[\frac{1}{2}\left\{\frac{V^2}{3} - \frac{7V^2}{3} - 2V^2\right\}\right]^{1/2}$$

$$= \left[\frac{1}{2}\left\{\frac{8V^2 - 6V^2}{3}\right\}\right]^{1/2} = \frac{V}{\sqrt{3}}$$

Form factor = $V/\sqrt{3}\,/\,V/2 = \dfrac{2}{\sqrt{3}} = 1.155$

20) To the circuit shown in fig. consisting of a 1k ohm resistor connected in series with a 50mH coil, a 10V rms, 10KHz signal is applied. Find i) impedance Z, ii) current I, iii) Phase angle θ & iv) Voltage across R & L.



SOL:

Inductive reactance $X_L = \omega L$

$$= 2\pi f L = (6.28)(10^4)(50 \times 10^{-3}) = 3140\ \Omega$$

In rectangular form,

Total impedance $Z = (1000 + j3140)\ \Omega$

$$= \sqrt{R^2 + X_L^2}$$

$$= \sqrt{(1000)^2 + (3140)^2} = 3295.4\ \Omega$$

AVRK

Current $I = V_S/Z = 10/3295.4 = 3.03$ mA

Phase angle $\theta = \tan^{-1}(X_L/R) = \tan^{-1}(3140/1000) = 72.33°$

Therefore, in polar form total impedance $Z = 3295.4 \angle 72.33°$

Voltage across resistance $V_R = IR = 3.03 \times 10^{-3} \times 1000 = 3.03$ V

Voltage across inductive reactaVnce $V_L = IX_L = 3.03 \times 10^{-3} \times 3140 = 9.51$ V

21) A sinusoidal voltage V=50 Sin ωt is applied to a series RL circuit. The current in the circuit is given by I=25 Sin (ωt-53^0). Determine (a) Apparent power (b) Power factor (c) Average power.

SOL:

(a) Apparent power $P = V_{eff} I_{eff}$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$= \frac{50 \times 25}{2} = 625\,\text{VA}$$

(b) Power factor = cos θ

where θ is the angle between voltage and current

$$\theta = 53°$$

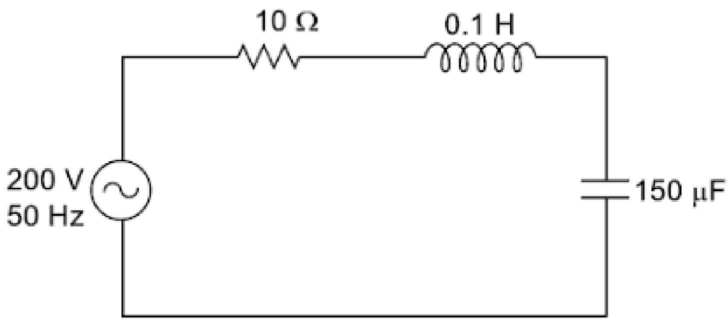
∴ power factor = cos θ = cos 53° = 0.6

(c) Average power $P_{av} = V_{eff} I_{eff} \cos\theta$

$$= 625 \times 0.6 = 375\ \text{W}$$

22) A coil of resistance 10Ω and an inductance of 0.1H is connected in series with a capacitor of capacitance 150μF across at 200V, 50 Hz supply. Calculate

i) Impedance ii) Current iii) Power & power factor of the circuit



SOL:

AVRK

(i) Total impedance

$$Z = R + j\omega L - \frac{j}{\omega c}$$

$$= 10 + j31.45 - j21.22$$

$$= 10 + j10.194$$

$$= 14.279 \angle 45.55$$

(ii) Current $I = \frac{V}{Z}$

$$= \frac{200\angle 0^\circ}{14.279\angle 45.55^\circ}$$

$$= 14\angle -45.55^\circ$$

(iii) Power factor $= \cos(45.55^\circ)$

$$= 0.7 \text{ lagging}$$

Real power $= VI\cos\theta$

$$= 200 \times 14 \times 0.7$$

$$= 1.9 \text{ kW}$$

Reactive power $= VI\sin\phi$

$$= 200 \times 14 \times \sin(-45.55)$$

$$= -1.998 \text{ KVAR}$$

"−1" Sign indicates that it absorbs the reactive power.

23) An AC circuit consists of a pure resistance of 10Ω and is connected across an AC supply of 230V, 50Hz. Calculate (a) Current (b) Power consumed (c) Power factor (d) write down the equations for voltage and current.

SOL:

$$R = 10\ \Omega, \quad V = 230\text{ V}, \quad f = 50\text{ Hz}$$

(a) Current

$$I = \frac{V}{R} = \frac{230}{10} = 23\text{ A}$$

(b) Power consumed

$$P = VI = 230 \times 23 = 5290\text{ W}$$

AVRK

(c) Power factor

Since the voltage and current are in phase with each other, $\phi = 0°$

$$pf = \cos\phi = \cos(0°) = 1$$

(d) Voltage and current equations

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 230 = 325.27\text{ V}$$

$$I_m = \sqrt{2}I = \sqrt{2} \times 23 = 32.53\text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s}$$

$$v = V_m \sin\omega t = 325.27 \sin 314.16\,t$$

$$i = I_m \sin\omega t = 32.53 \sin 314.16\,t$$

24) An inductive coil having negligible resistance and 0.1H inductance is connected across an AC supply of 220V, 50Hz. Calculate (a) Inductive reactance (b) RMS value of Current (c) Power consumed (d) Power factor (e) write down the equations for voltage and current.

SOL:

$$L = 0.1\text{ H},\quad V = 200\text{ V},\quad f = 50\text{ Hz}$$

(a) Inductive reactance

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$$

(b) rms value of current

$$I = \frac{V}{X_L} = \frac{200}{31.42} = 6.37\text{ A}$$

(c) Power

Since the current lags behind the voltage by 90° in purely inductive circuit, $\phi = 90°$

$$P = VI\cos\phi = 200 \times 6.37 \times \cos(90°) = 0$$

(d) Power factor

$$pf = \cos\phi = \cos(90°) = 0$$

(e) Equations for voltage and current

$$V_m = \sqrt{2}\,V = \sqrt{2} \times 200 = 282.84\text{ V}$$

$$I_m = \sqrt{2}\,I = \sqrt{2} \times 6.37 = 9\text{ A}$$

$$\omega = 2\pi f = 2\pi \times 50 = 314.16\text{ rad/s}$$

$$v = V_m \sin\omega t = 282.84 \sin 314.16\,t$$

$$i = I_m \sin\left(\omega t - \frac{\pi}{2}\right) = 9\sin\left(314.16\,t - \frac{\pi}{2}\right)$$

25) A capacitor has a capacitance of 30μF which is connected across an AC supply of 230V, 50Hz. Calculate (a) capacitive reactance (b) RMS value of Current (c) Power consumed (d) Power factor (e) write down the equations for voltage and current.

SOL:

$$C = 30\ \mu\text{F},\quad V = 230\text{ V},\quad f = 50\text{ Hz}$$

(a) Capacitive reactance

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 30 \times 10^{-6}} = 106.1\ \Omega$$

(b) rms value of current

$$I = \frac{V}{X_C} = \frac{230}{106.1} = 2.17\ \text{A}$$

(c) Power
Since the current leads the voltage by 90° in purely capacitive circuit, $\phi = 90°$

$$P = VI\cos\phi = 230 \times 2.17 \times \cos(90°) = 0$$

(d) Power factor

$$\text{pf} = \cos\phi = \cos(90°) = 0$$

(e) Equations for voltage and current

$$V_m = \sqrt{2}\ V = \sqrt{2} \times 230 = 325.27\ \text{V}$$
$$I_m = \sqrt{2}\ I = \sqrt{2} \times 2.17 = 3.07\ \text{A}$$
$$\omega = 2\pi f = 2\pi \times 50 = 314.16\ \text{rad/s}$$
$$v = V_m \sin\omega t = 325.27 \sin 314.16\, t$$
$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right) = 3.07 \sin\left(314.16\, t + \frac{\pi}{2}\right)$$

26) In a series circuit containing resistance and inductance, the current and voltage are expressed as $i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right)$ and $v(t) = 20\sin\left(314t + \frac{5\pi}{6}\right)$.

(a) What is the impedance of the circuit?
(b) What are the values of resistance, inductance & power factor?
(c) What is the average power drawn by the circuit?

SOL:

$$i(t) = 5\sin\left(314t + \frac{2\pi}{3}\right), \quad v(t) = 20\sin\left(314t + \frac{5\pi}{6}\right)$$

(a) Impedance

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{20}{5} = 4\ \Omega$$

(b) Power factor, resistance and inductance
Current $i(t)$ lags behind voltage $v(t)$ by an angle $\phi = 150° - 120° = 30°$

$$\text{pf} = \cos\phi = \cos(30°) = 0.866\ \text{(lagging)}$$
$$\mathbf{Z} = 4\angle 30° = 3.464 + j2\ \Omega$$
$$R = 3.464\ \Omega$$
$$X_L = 2\ \Omega$$
$$X_L = \omega L$$
$$2 = 314 \times L$$
$$L = 6.37\ \text{mH}$$

(c) Average power

$$P = VI\cos\phi = \frac{20}{\sqrt{2}} \times \frac{5}{\sqrt{2}} \times 0.866 = 43.3\ \text{W}$$

AVRK